#1 Non-renewable Resource Extraction and the Right to Sell

a) The P.V. max problem is:

$$\max_{\{R_0, R_1\}} V_0 = p_0 R_0 - c(R_0) + \beta\left[p_1 R_1 - c(R_1)\right]$$

$$\text{s.t. } R_0 + R_1 \leq S_0 .$$

→ I assume initially that the resource constraint is binding. This will have to be checked as plausible later with the numbers provided.

$\Rightarrow \quad R_1 = S_0 - R_0 .$

$$\max_{R_0} V_0 = p_0 R_0 - c(R_0) + \beta\left[p_1 (S_0 - R_0) - c(S_0 - R_0)\right]$$

$$\Rightarrow \frac{\partial V_0}{\partial R_0} = 0 \Rightarrow p_0 - c'(R_0) = \beta\left[p_1 - c'(R_1)\right]$$

$$\Rightarrow \frac{p_1 - c'(R_1) - \left[p_0 - c'(R_0)\right]}{p_0 - c'(R_0)} = r$$

The marginal rent increases at rate r.

b) $p_0 = p_1 = 50,\ S_0 = 500,\ r = 0.1,\ c(R_t) = R_t^2/20.$

$c'(R_t) = R_t/10.$

Introducing into the F.O.C., we get

$$1.1\left(50 - \frac{R_0^*}{10}\right) = 50 - \left(\frac{500 - R_0^*}{10}\right)$$

$$\Rightarrow R_0^* = 262 \ \& \ R_1^* = 238.$$

$$\left.\begin{aligned} P_0 &\equiv p - C'(R_0^*) = 50 - 26.2 = 23.8 > 0 \\ P_1 &\equiv p - C'(R_1^*) = 50 - 23.8 = 26.2 > 0 \end{aligned}\right\} \text{RESOURCE CONSTRAINT IS BINDING.}$$

$\Rightarrow \dfrac{P_1 - P_0}{P_0} = 10\%$ The marginal rent increases at a rate of 10%.

c) $S_0 = 1200$:

$$1.1\left(50 - \frac{R_0}{10}\right) = 50 - \left(\frac{1200 - R_0}{10}\right)$$

$\Rightarrow R_0 = 595.2 \Rightarrow p - C'(R_0) < 0.$

The firm is making losses from the last units extracted.

$\Rightarrow$ The stock constraint is not binding.

$\Rightarrow p = C'(R_0^*) \Rightarrow 50 = \dfrac{R_0^*}{10} \Rightarrow \boxed{R_0^* = 500}$

Similarly $R_1^* = 500$.

$R_0^* + R_1^* = 1000 < S_0$. Given the increasing marginal cost of extraction, the firm prefers not to extract all the oil. The high marginal cost of extraction do not lower the selling price passed 500 units.

d) Since the oil deposit cannot be sold, it makes no difference to anyone to leave 200 units for period 2. The resource constraint becomes $S_0 = 1000$ and it is still not binding. The extraction rate remains at 500 units per period.

e) If the deposit can be sold, then the owner will choose R_t in order to maximize the P.V. for $t = 0, 1, 2$.

$$\Rightarrow \max_{R_0, R_1, R_2} V_0 = \sum_{t=0}^{2} \beta^t \left[p_t R_t - c(R_t) \right]$$

$$\text{s.t.} \quad \sum_{t=0}^{2} R_t = 1200.$$

We have:

$$50 - \frac{R_0}{10} = \frac{1}{1.1}\left(50 - \frac{R_1}{10}\right) = \left(\frac{1}{1.1}\right)^2 \left(50 - \frac{R_2}{10}\right)$$

$$R_0 + R_1 + R_2 = 1200$$

3 eqtns, 3 unknowns: $R_0^* = 409.4$, $R_1^* = 400.3$
$R_2^* = 390.3$

Since $p - C'(R_t) > 0 \ \forall t$, the resource constraint is binding.
The owner can sell his oil deposit at its present value at any time and thus receive the maximum value for the deposit.

Note that introducing the possibility of selling the resource is quite "environmentally friendly" and good for the future generations. The owner will leave more than 200 barrels for the future generation. A right to sell induces owners to include the demand from future generations into their extraction decisions.