Renewable Resource with Existence Value

(1) $$\max \; J = \int_0^{\infty} e^{-\delta t}\left\{(p - c(x_t))\,h_t + V(x_t)\right\} dt$$

$$\text{s.t. } \dot{x} = F(x) - h_t$$

$$H_c = (p - c(x))\,h_t + V(x_t) + \mu_t\left(F(x) - h_t\right)$$

This is the current-value Hamiltonian.

$$\frac{\partial H_c}{\partial h} = p - c(x_t) - \mu_t = 0 \Rightarrow \boxed{\mu_t = p - c(x_t)} \; [A]$$

$$\dot{\mu}_t - \delta \mu_t = -\frac{\partial H_c}{\partial x_t} = h_t c'(x_t) - V'(x_t) - \mu_t F'(x_t)$$

$$\Rightarrow \boxed{\frac{\dot{\mu}_t}{\mu_t} = \delta + \frac{h_t c'(x_t) - V'(x_t)}{\mu_t} - F'(x_t)} \; [B]$$

μ_t denotes the shadow price the resource capital x_t. It represents the marginal contribution of x_t on the objective function in current-value terms.

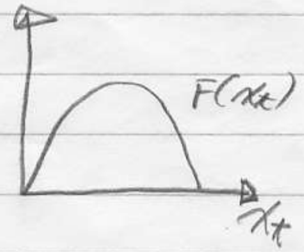
Condition [A] requires that along the optimal path, at the margin, a unit of the resource harvested must not contribute less to instantaneous profits than the shadow value of x_t, otherwise it would be better not to harvest it. And conversely if it contributes more.

Equation [B] says that one must be indifferent between leaving the last unit unharvested or harvesting it and investing the proceeds to get $\mu_t \delta$. The foregone profits from the last unit are: the appreciation $\dot{\mu}_t$; plus the existence value $V'(x_t)$; plus the total cost reduction $h_t c'(x_t)$; plus the value of the change in the natural growth rate of the resource $\mu_t F'(x_t)$.

(2) In the steady state, we must have $\dot{\mu}_t = \dot{x}_t = 0$. Hence $h_t = F(x_t)$.

$$\Rightarrow F'(x) = \delta + \frac{F(x)c'(x) - V'(x)}{p - c(x)} \qquad [C]$$

The second term on the RHS is negative. Hence $F'(x_t) < \delta$ along the optimal path. Adding $V'(x_t)$ on the RHS reduces the optimal value of $F'(x)$. Since $F''(x)$ is negative, this implies that an existence value increases the steady state resource stock.



(4) A non-renewable resource implies $F(x) = 0$. The steady-state condition becomes:

$$\frac{V'(x)}{p - c(x)} = \delta \text{ or } \frac{V'(x)}{\delta} = p - c(x)$$

$V'(x)/\delta$ represents the present value of existence benefits from a marginal unit of resource stock.

$p - c(x)$ is the instantaneous gain from that marginal unit.

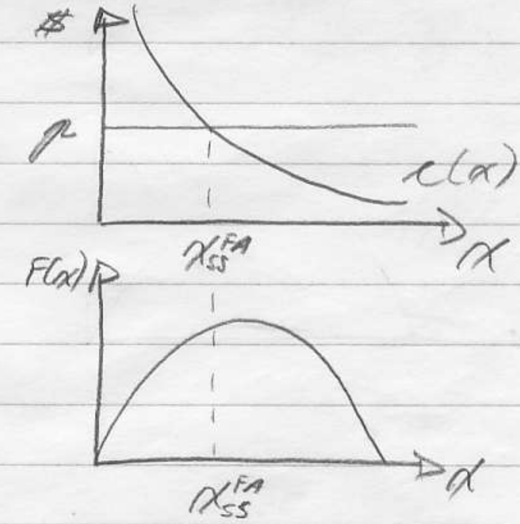
One should stop extracting before $V'(x)/\delta$ falls below $p - c(x)$. This corresponds to a steady state with no harvesting.

(3) According to relation [C], a very large $V'(x)$ implies that $-F'(x)$ must be maximized. This is achieved at $F(x) = 0$, i.e. the resource is left unexploited.

(5) With free access and an arbitrarily large number of exploiters, all rents must be exhausted. Since there are positive rents with $p > c(x_t)$ and unit costs do not depend on the harvest rate h_t, the stock of the resource will immediately jump down to x_{ss} such that

$$p = c(x_{ss}^{FA})$$

with $h_{ss}^{FA} = F(x_{ss}^{FA})$.



Increasing the discount rate has no effect on the free access equilibrium.