3. Private ownership with costly exclusion of poachers

As discussed briefly in class, one should make a difference between the right to exclude and the real ability to do so. This exercise considers the case of a single owner of a resource who decides on how to exploit the resource but must pay to exclude potential trespassers, referred to as poachers here.

Poaching can be viewed as a *sequential* game between a resource owner and n poachers. In the first stage, the owner decides on the number of hours of labor he will hire ($L \geq 0$) to exploit the resource, say a fishery, and on the intensity with which he monitors poaching. As a result of this policing, each poacher expects to be caught with probability $\lambda \in (0, 1)$. If the owner catches a poacher, he confiscates his catch but can exact no other penalty.

We initially *restrict attention* to the second stage where the n poachers choose the number of hours of illegal activity simultaneously, after *observing* both L and λ. Assume that each poacher wishes to maximize his expected gain. Each poacher has T hours per day to work and can divide them between legal work and poaching. Legal work pays w per hour and the stolen catch sells for p per unit. If player i poaches for h_i hours, he earns in expectation:

$$\lambda w(T - h_i) + (1 - \lambda)\left(w(T - h_i) + \frac{h_i}{h_i + h_{-i} + L} pF(h_i + h_{-i} + L)\right),$$

where $F(\cdot)$ is the total output function, $F'(\cdot) > 0$, $F''(\cdot) < 0$, and $h_{-i} = \sum_{j \neq i} h_j$.

(1) Find the symmetric Nash equilibrium conditions for any given pair (L, λ). (Consider only the interior conditions, i.e. $h_i^* < T$.)

(2) Assume now that poaching is *organized* by a criminal gang that controls the number of poachers in order to maximize their total expected profits. Characterize the equilibrium condition for h_i, $i = 1, ..., n$, in this case. Compare with your result in (1) and comment.

For the rest of this question, we assume unorganized poaching, as in (1).

(3) Denote the aggregate poaching hours as $H(L, \lambda) = \sum_{i=1}^{n} h_i(L, \lambda)$. Verify that H is strictly decreasing in L and λ for $H > 0$.

(4) For fixed L and λ, characterize the free-access limit case where $n \to \infty$. What is the equilibrium value of the average product of the resource? Interpret.

(5) Let us now turn to the owner's problem while assuming that n is very large, i.e. the case where $n \to \infty$. For fixed policing λ, derive the owner's first-order conditions for L (don't forget that he is a first-mover). What is the equilibrium poaching level H induced by the owner's choice? Interpret.

ANSWERS

(1) **Symmetric NE conditions:**

Given (L, λ), the poaching NE is found by deriving each poachers' reaction function, i.e. its FOCs for h_i:

$$\max_{h_i} J_i = \lambda w(T - h_i) + (1 - \lambda)\left(w(T - h_i) + \frac{ph_i}{X}F(X)\right)$$

where $X = h_i + h_{-i} + L$. We have:

$$\frac{\partial J_i}{\partial h_i} = -w + p(1-\lambda)\left\{\frac{F(X)}{X} + h_i\frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right)\right\} = 0.$$

This FOC says that increasing poaching time by one marginal unit has three effects: i) lower legitimate income by w; ii) increase expected income from poaching by $p(1-\lambda)F(X)/X$, which represents the value of average product adjusted for the probability of being caught; and iii) lower the productivity for existing poaching efforts h_i due to the decrease in value of average product and adjusted for the probability of being caught, i.e. $p(1-\lambda)h_i\frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right)$.

For a symmetric NE, we let $h_1 = h_2 = ... = h_n \equiv h^*$. Define $X^* = nh^* + L = H^* + L$, where $H^* = nh^*$. We have:

$$p(1-\lambda)\left\{\frac{F(X^*)}{X^*} + h^*\frac{\partial}{\partial X}\left(\frac{F(X^*)}{X^*}\right)\right\} = w.$$

This characterizes the poaching NE for given (L, λ). It is a NE between poachers because each poacher is maximizing his expected gain given what the others are doing.

(2) **Organized crime**

With organized crime, we can assume that a gang leader has full control over entry to the poaching ground. He can thus maximize the total gains from poaching. The problem for this leader can be expressed as follows:

$$\max_{h_1,h_2,\ldots,h_n} W = \sum_{i=1}^{n} \lambda w(T - h_i) + (1 - \lambda)\left(w(T - h_i) + \frac{ph_i}{X}F(X)\right),$$

which implies,

$$\frac{\partial W}{\partial h_i} = -w + p(1-\lambda)\left\{\frac{F(X)}{X} + \sum_{i=1}^{n} h_i\frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right)\right\} = 0.$$

Note the main difference with the FOC of the single poacher. Letting $h_1 = h_2 = ... = h^o$, we get

$$p(1-\lambda)\left\{\frac{F(X^o)}{X^o} + nh^o\frac{\partial}{\partial X}\left(\frac{F(X^o)}{X^o}\right)\right\} = w.$$

The effect of a change in average product is now multiplied by nh instead of just h. *The criminal gang resolves the problem of the commons between poachers!* This leads to less over-exploitation of the resource. Within our framework here, the owner of the resource prefers a centralized poaching organization over unorganized poaching.

NB An alternative way to solve the problem is to simply assume that the leader of the gang chooses the total amount of poaching effort H to solve:

$$\max_H W = w(nT - H) + (1-\lambda)\left(w(nT-H) + \frac{pH}{X}F(X)\right),$$

which yields the same result.

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(3) **Effects of L and λ on H^*:**

From the NE condition for poaching, we have:

$$\psi(H; L, \lambda) \equiv \frac{F(X)}{X} + \frac{H}{n}\frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right) - \frac{w}{p(1-\lambda)} = 0.$$

Using the implicit function theorem, we have:

$$\frac{\partial H}{\partial L} = -\frac{\psi_L}{\psi_H}$$

where

$$\psi_L = \frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right) + \frac{H}{n}\frac{\partial^2}{\partial X^2}\left(\frac{F(X)}{X}\right)$$

and

$$\psi_H = \frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right) + \frac{H}{n}\frac{\partial^2}{\partial X^2}\left(\frac{F(X)}{X}\right) + \frac{1}{n}\frac{\partial}{\partial X}\left(\frac{F(X)}{X}\right).$$

Assuming that the average product curve is not too convex, i.e. $\frac{\partial^2}{\partial X^2}\left(\frac{F(X)}{X}\right)$ is not "too positive", we obtain

(1) $$\frac{\partial H}{\partial L} < 0,$$

since $\psi_L < 0$ and $\psi_H < 0$.

Similarly,

$$\frac{\partial H}{\partial \lambda} = -\frac{\psi_\lambda}{\psi_H}$$

where $\psi_\lambda = -\frac{w\lambda}{p(1-\lambda)} < 0$ and hence

(2) $$\frac{\partial H}{\partial \lambda} < 0.$$

From (1) and (2) we observe that the owner can drive out poaching through two channels: the direct one is to increase enforcement λ, but he can also do it indirectly by hiring more labor which reduces the poaching payoff.

(4) **Let $n \to \infty$**
As n becomes arbitrarily large, the effect of a chnage in the average product vanishes at the individual level, i.e. $h^* \to 0$ or $H^*/n \to 0$, just as in the tragedy of the commons case. The equilibrium condition becomes:

$$p(1-\lambda)\frac{F(X^*)}{X^*} = w. \tag{3}$$

Note that this equation is the same as with the tragedy of the commons, i.e. the poachers' rents disappear as $n \to \infty$., with the difference that the average gain must be adjusted for the probability of being caught and punished.

(5) **Owner's choice of labor when n is arbitrarily large:**
Assume that the unit cost of labor is also w. The owner's problem is, for fixed λ:

$$\max_L \pi = p\frac{F(X)}{X}L - wL,$$

where

$$p\frac{F(X)}{X} = \frac{w}{1-\lambda} \text{ if } H > 0.$$

Hence, as long as the poaching effort is positive, the problem is:

$$\max_L \pi = \frac{w}{1-\lambda}L - wL.$$

This implies that

$$\frac{\partial \pi}{\partial L} = \frac{w}{1-\lambda} - w > 0, \forall L.$$

We have a corner solution! With an arbitrarily large number of poachers, the average product remains constant as L increases. The owner will consequently hire labor up to the point where $H = 0$, i.e. encroachers loose interest in poaching due to its low average productivity.

We could have reached the same conclusion by noting from equation (3) that $\partial H/\partial L = -1$. This implies that each time the owner hires unit of work, one unit of poaching is deterred. The average product is thus not affected. Hence, if the owner makes positive profits, as should be the case to have an owner in the first place, then average product is above the wage rate and it pays for the owner to hire this additional worker. This is only valid as long as the amount of poaching is strictly positive.