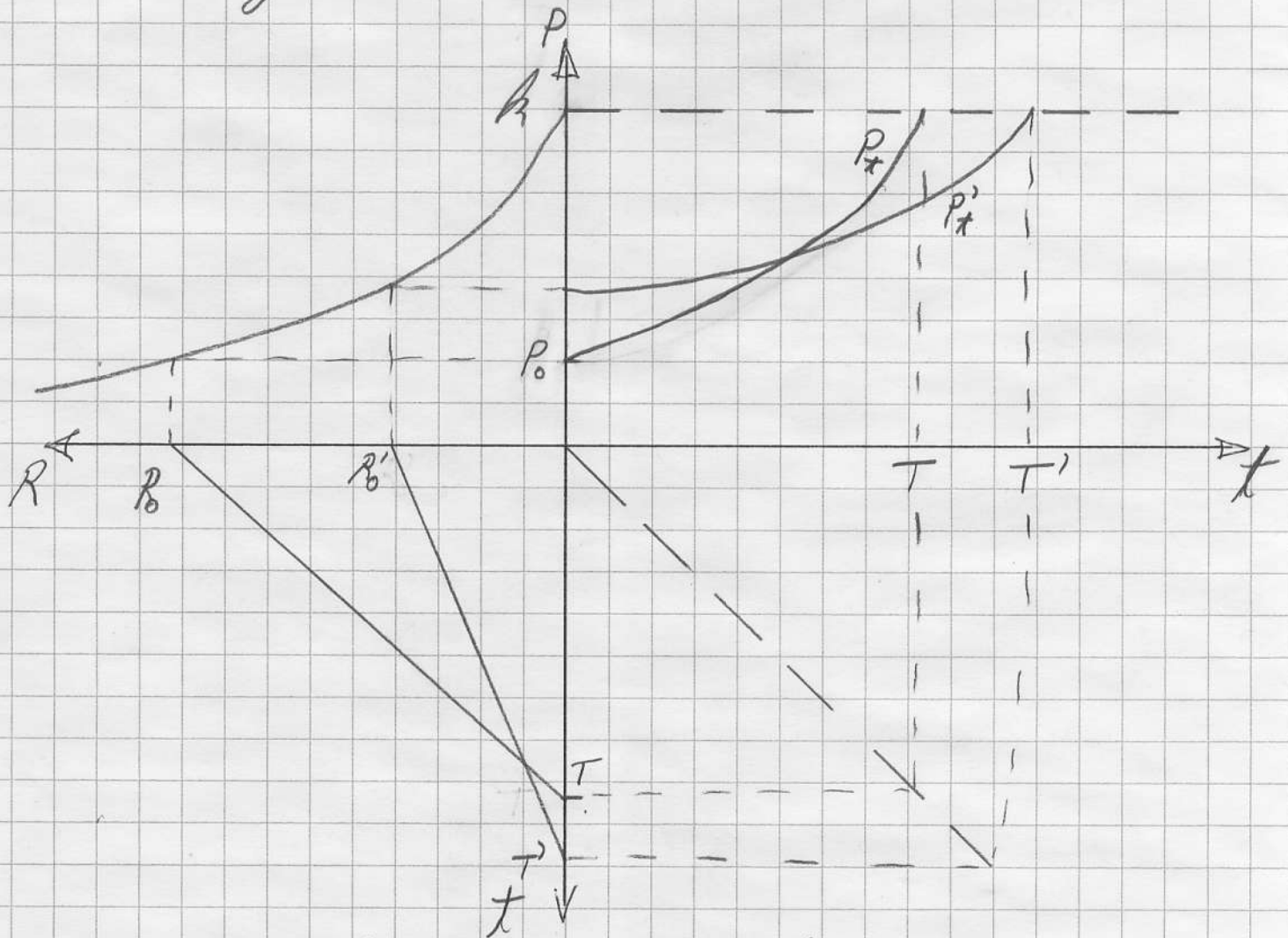
Oil Depletion, Global Warming, and Kyoto.

a) A unit tax on oil has the same effect as increasing the constant marginal cost.



Let P_t ≡ initial price path
and P_t' ≡ price path with tax.

$\frac{\dot{P}_t}{P_t} = r$ with zero marginal cost.

With a tax q, we will have

$$\frac{\frac{\partial}{\partial t}(P_t' - q)}{P_t' - q} = \frac{\dot{P}_t'}{P_t' - q} = r$$

For all $P_t = P_t'$, we have

$$\dot{P}_t = r P_t > \dot{P}_t' = r P_t - rq.$$

Hence, if $P_0' \leq P_0$, $P_t' < P_t$, $\forall t > 0$.

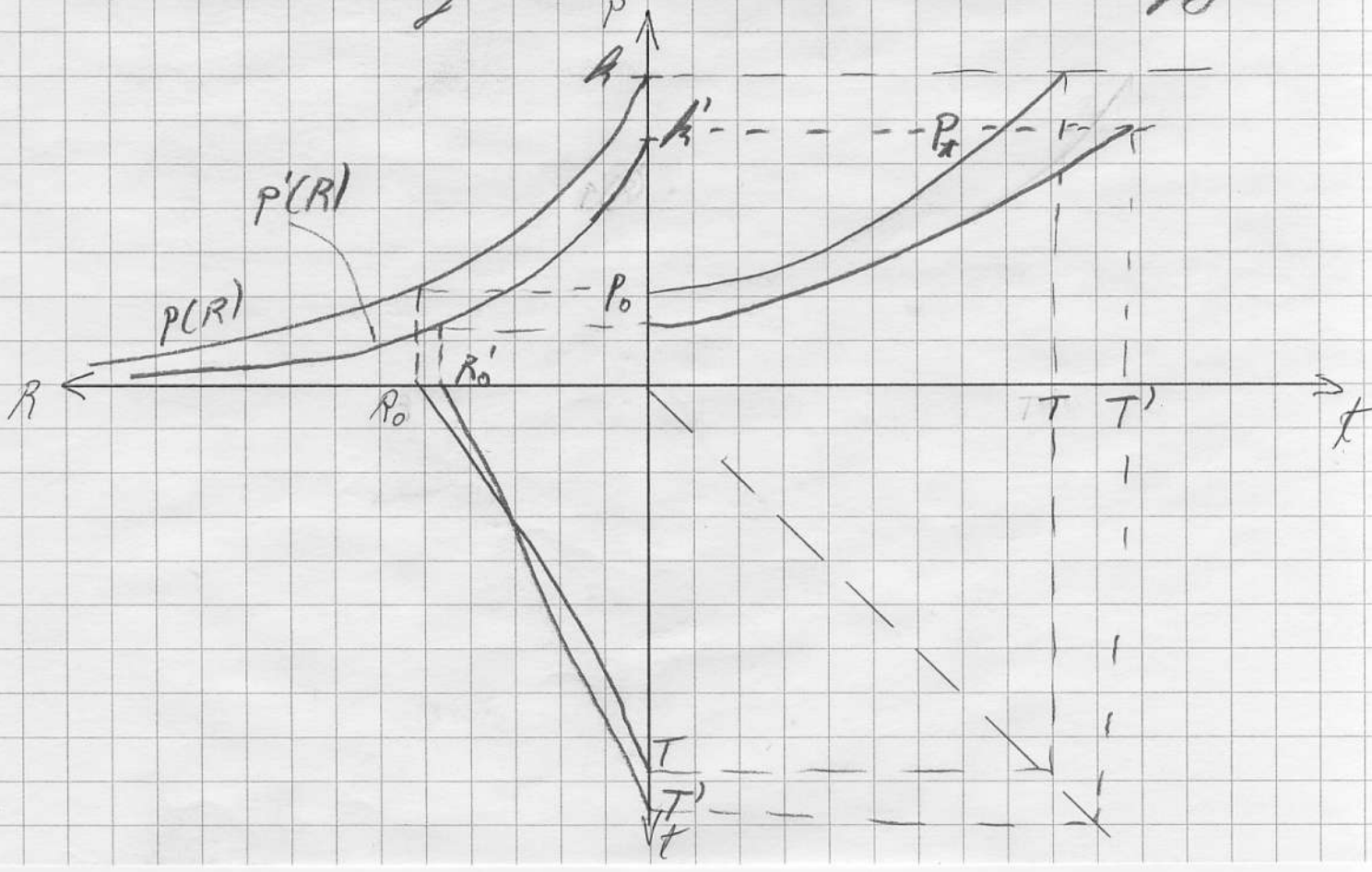
The resource is depleted before reaching the choke price.

Therefore, $P_0' > P_0$. And in order to use up all of the resource, we must have:

$$T' > T,$$

i.e. the new price path crosses the initial one and it takes longer to deplete the resource.

b) A subsidy on alternative energy:



Assume that the demand curve for oil shifts down to $P'(R)$ because of the subsidy.

We still have $\frac{\dot{P}_t'}{P_t'} = \frac{\dot{P}_t}{P_t} = r$.

If $P_0' \geq P_0$, then $R_t' < R_t \; \forall t > 0$.

The resource stock is not completely depleted.

Hence, $P_0' < P_0$, $P_t' < P_t \; \forall t$, and $T' > T$, since the price paths cannot cross each other.

With the subsidy, the resource takes longer to be depleted. Note that this does not mean that oil output is always below the initial one as it will be larger at the end.

c) i) The effects identified in a) and b) are both present simultaneously, i.e. $\dot{P}_t' < P_t$ for $P_t' = P_t$ due to tax effect. The demand schedule decreases over time. Clearly, the resource

will be depleted in longer time