Solution to
Grazing land as a renewable resource

The production function for *annual* beef production on a pasture is given by

$$B_t = G_t b(H_t), \quad \text{with } b_H > 0,\ b(0) = 0,\ \lim_{H_t \to \infty} b(H_t) = 1$$

where B_t, G_t, and H_t respectively denote the beef produced (tons), the amount of grazable grass (tons), and the number of beef cattle (heads) allowed to graze. The amount of grass available in year $t+1$ depends solely on the number of beef cattle used the preceding year, i.e.

$$G_{t+1} = g(H_t),$$

with $g'(H_t) < 0$, $g(0) = G_0$, $\lim_{H_t \to \infty} g(H_t) = 0$. Beef sells at a price of p per ton, and the cost of herding, transporting, and processing *each* head of cattle is c.

(1) Given G_t, *characterize* the conditions for

(a) the maximum beef production.

Since $b_H > 0$, maximum production is obtained by grazing all the grass with an infinitely large number of cattle heads.

(b) the number of cattle that will maximize year t's profits.

Year t's profits are maximized by solving

$$\max_{H_t} \pi_t = pG_t b(H_t) - cH_t.$$

This yields the following FOC:

$$\frac{\partial \pi_t}{\partial H_t} = pG_t b'(H_t) - c = 0.$$

or

$$b'(H_t) = \frac{c}{pG_t}.$$

This completely characterizes the choice of H_t to maximize one-period profits given G_t. It simply accounts for the unit cost of a head of cattle. With $b'' < 0$, this implies that higher prices of beef and more grass will lead to more heads of cattle, while the converse holds with higher unit cost.

(c) The *open access* number of cattle.

With open access, we expect to see complete rent exhaustion, which is obtained when average product is equal to the unit cost of cattle, given G_t, i.e.

$$\frac{pG_t b(H_t)}{H_t} = c$$

OR

$$\frac{b(H_t)}{H_t} = \frac{c}{pG_t}.$$

Interpret briefly your results. (We assume that the second-order conditions for a maximum are always satisfied.)

(2) Characterize the conditions for a steady-state equilibrium herd size under open access, year-to-year profit maximization, and maximum sustainable yield.

In a steady state equilibrium, we must have $G_{t+1} = G_t$, which implies that $G_t = g(H_t), \forall t$.

Inserting this into the open access condition, we get

$$\frac{pg(H_t)b(H_t)}{H_t} = c.$$

In the case of year-to-year profit maximization, we get

$$pg'(H_t)b(H_t) + pg(H_t)b'(H_t) = c.$$

As for the maximum sustainable yield, the following problem must be solved:

$$\max_{H_t} g(H_t)b(H_t).$$

The FOC of which is:

$$g'(H_t)b(H_t) + g(H_t)b'(H_t) = 0.$$

Note the large difference between the MSY solution and the one-period maximum production solution. The latter does not account for the fact that maximizing today's output will lead to no output next year. The MSY solution accounts for the effect of increasing the number of heads today on the reduced productivity next year.