1. Free access and regulation (Based on DH (1979).)

We wish to further analyze the problem of free access to a fishery with n identical fishers seen in class. We consider still the symmetrical Nash equilibrium. (NB Provide an intuitive explanation for each answer.)

(1) Find the *tax rate* per unit of effort that would re-establish optimality. What does it represents exactly?
(2) How does the unit tax value vary with the number of firms?
(3) Do fishing firms prefer the free access regime with the tax or without? Analyze as completely as possible.
(4) Assume instead that the government chooses to distribute licenses to fishing firms. What will be the equilibrium price of each license?
(5) Do firms prefer the license to the tax?
(6) Discuss whether those two regulation instruments really change anything fundamental to the initial problem of free access.

ANSWERS

(1) **A unit tax**

The objective is to find a tax rate that can re-establish the efficient number of boats on the fishery. In other words, with the tax, the total number of boats X must respect:

$$pf'(X^*) = w \text{ where } X^* = nx^*. \tag{1}$$

Let t be the tax per boat. The new NE is now characterized by (see class notes):

$$pf'(X^t) = (w+t) + \frac{n-1}{n} pX^t \frac{\partial}{\partial X}\left(\frac{f(X^t)}{x^t}\right). \tag{2}$$

If we set t at the following value:

$$\begin{aligned} t^* &= -\frac{n-1}{n} pX^* \frac{\partial}{\partial X}\left(\frac{f(X^*)}{X^*}\right), & (3)\\ &= -(n-1)px^* \frac{\partial}{\partial X}\left(\frac{f(X^*)}{X^*}\right), & (4) \end{aligned}$$

then the tax equilibrium becomes the one sought for in equation (1).

Note that X^* here is a given value defined in equation (1). Hence, the optimal tax t^* is actually equal to the **value of the externalities** that one firm imposes on the others when it decides to send another boat on the fishery. Such a tax is referred to as a **Pigovian tax** in the literature, which is defined as the marginal external damage imposed on others *at the efficient level*. It is related to the **Polluter-Pay-Principle**. It assumes that the regulator knows what the efficient input level must be.

(2) **Tax and number of firms**
According to expression (4), the tax increases with the number of firms. This is not surprising, as we have seen that the larger the number of firms, the larger the value of the externalities that each firm imposes on others. When $n = 1$, this value is nil; when $n \to \infty$, this value is maximized.

(3) **Firm profits with and without the tax** (NB This answer is based on DH chap 3 section 4.)
We wish to compare total firm profits with the tax and without. The answer is not *a priori* obvious because even though firms must pay taxes, higher efficiency with taxes means that there are more total rents. Its is thus a question of comparing the effect of a larger pie versus a lower share of the pie.

Let R be the total *net* rents received by the firms. In the **no-tax NE**, we have:

$$\begin{aligned} R^e &= pf(X^e) - wX^e, && (5) \\ &= \left(p\frac{f(X^e)}{X^e} - w\right) X^e, && (6) \\ &= \frac{p}{n}\left(f(X^e) - X^e f'(X^e)\right). && (7) \end{aligned}$$

The last equation is obtained by making use of the NE condition seen in class.

In the **NE with tax**, we have:

$$\begin{aligned} R^t &= pf(X^*) - (w + t^*)X^*, && (8) \\ &= \left(p\frac{f(X^*)}{X^*} - (w + t^*)\right) X^*, && (9) \\ &= \frac{p}{n}\left(f(X^*) - X^* f'(X^*)\right). && (10) \end{aligned}$$

Now it must be the case that $R^t < R^e$ because $X^e > X^*$ and

$$\frac{\partial}{\partial X}\left(f(X) - Xf'(X)\right) = -Xf''(X) > 0. \qquad (11)$$

Hence, in the absence of redistribution of tax receipts, firms prefer the no-tax NE equilibrium.

(4) **License price**

We assume that the government somewhat *arbitrarily* distributes the number X^l of licenses to operate a boat on the fishery and that those licenses can thereafter be exchanged between firms. A market for licenses thus develops and we are looking for its equilibrium price. To simplify, let us assume that firms are price-takers in this market for licenses (as proposed in DH 67) and denote the price as q. This essentially means that the *opportunity cost* of a boat is now $w+q$ instead of just w in the free-access NE.

In equilibrium, it must be the case that given the license price q, each firm is satisfied with the number of licenses that it has, i.e. it cannot increase its profits by selling or acquiring more licenses. More formally, the marginal profit from a license must be nil. The profit for one firm is:

$$\pi_i = x_i p \frac{f(X)}{X} - (w+q)x_i \text{ where } X = x_{-i} + x_i. \tag{12}$$

Hence, the zero marginal profit condition is:

$$\frac{\partial \pi_i}{\partial x_i} = p\frac{f(X)}{X} + x_i p \frac{\partial}{\partial X}\left(\frac{f(X)}{X}\right) - (w+q) = 0. \tag{13}$$

The price q must be such that this condition is satisfied for all firms. Assuming identical firms, we have:

$$p\frac{f(X)}{X} + xp\frac{\partial}{\partial X}\left(\frac{f(X)}{X}\right) - (w+q) = 0. \tag{14}$$

This implies

$$pf'(X) - w - q + \left(\frac{n-1}{n}\right)\left(p\frac{f(X)}{X} - pf'(X)\right) = 0. \tag{15}$$

Now if we assume that the authorities will distribute the efficient number X^* of licenses, we have $pf'(X^*) - w = 0$, such that:

$$q^* = p\left(\frac{n-1}{n}\right)\left(\frac{f(X^*)}{X^*} - f'(X^*)\right), \tag{16}$$

$$= -p\left(\frac{n-1}{n}\right)\frac{\partial}{\partial X}\left(\frac{f(X^*)}{X^*}\right). \tag{17}$$

This price is equal to the optimal tax rate.

(5) **Licenses v. taxes**

If firms must buy their licenses form the government, then there is obviously no difference between the tax and the license equilibrium as far as their rents are concerned. If the licenses are distributed freely to the firms, then they prefer the license scheme.

(6) The problem with the introduction of those instruments is that they *implicitly* assume that boats can be excluded if firms do not pay the tax or the licenses. If such exclusion can be achieved, then the problem of free access did not really exist in the first place. By introducing taxes or licenses, we are really only determining who pays what to enter the fishery, but the real problem boils down to one of exclusion. We could also have simply chosen who can and cannot enter the fishery.