2. Discrete-time Analysis of a Fishery

a) P.V. maximization:

$$\max_{H_t, E_t, S_t} V_0 = \sum_{t=0}^{\infty} \beta^t \left[p H_t - \mu E_t \right]$$

$$\text{s.t.} \quad H_t = e E_t S_t$$

$$S_{t+1} = S_t + G(S_t) - H_t$$

let $H_t = G(S_t) - (S_{t+1} - S_t) \equiv H(S_{t+1}, S_t)$

$$E_t = \frac{H_t}{e S_t} = \frac{H(S_{t+1}, S_t)}{e\, S_t} \equiv E(S_{t+1}, S_t)$$

$$\Rightarrow \max_{\{S_t\}} V_0 = \sum_{t=0}^{\infty} \beta^t \left[p H(S_{t+1}, S_t) - \mu E(S_{t+1}, S_t) \right]$$

F.O.C. $\Rightarrow$
$$\frac{\partial V_0}{\partial S_t} = \beta^t \left[p \frac{\partial H_t}{\partial S_t} - \mu \frac{\partial E_t}{\partial S_t} \right] + \beta^{t-1} \left[p \frac{\partial H_{t-1}}{\partial S_t} - \mu \frac{\partial E_{t-1}}{\partial S_t} \right] = 0$$

We have:

$$\frac{\partial H_t}{\partial S_t} = G'(S_t) + 1 \quad ; \quad \frac{\partial H_{t-1}}{\partial S_t} = -1$$

$$\frac{\partial E_t}{\partial S_t} = \frac{(G'(S_t)+1) e S_t - e H_t}{(e S_t)^2} \quad ; \quad \frac{\partial E_{t-1}}{\partial S_t} = \frac{-1}{e S_{t-1}}$$

Substitute into F.O.C. to get:

$$\beta\left\{p\left[1+G'(S_t)\right] - w\left[\frac{1+G'(S_t)}{eS_t} - \frac{H_t}{eS_t^2}\right]\right\}$$

$$= p - \frac{w}{eS_{t-1}}$$

In Steady-State, we have:
$H_t = G(S)$ and $S_t = S_{t-1} \equiv S$

$$\Rightarrow \beta\left\{p\left[1+G'(S)\right] - w\left[\frac{1+G'(S)}{eS} - \frac{G(S)}{eS^2}\right]\right\} = p - \frac{w}{eS}$$

$$\Rightarrow \frac{\left(p - \frac{w}{eS}\right)G'(S) + \frac{w}{eS^2}G(S)}{p - \frac{w}{eS}} = r$$

$$\Rightarrow \boxed{G'(S) = r - \frac{\frac{w}{eS^2}G(S)}{p - \frac{w}{eS}}}$$

This eqtn characterizes the S-S stock of the resource.
Note that $G'(S) < r$.

b) Effect of a change in productivity:

Let $\Psi(e,S) \equiv \left(p - \frac{w}{eS}\right)(G'(S) - r) + \frac{w}{eS^2} G(S)$

$$\frac{\partial S}{\partial e} = -\frac{\Psi_e}{\Psi_S}$$

$\Psi_e = \frac{w}{e^2 S}(G'(S) - r) - \frac{w}{e^2 S^2} G(S) < 0$ since $r > G'(S)$ in equil.

$$\Psi_S = \underbrace{\frac{w}{eS^2}(G'(S) - r)}_{(-)} + \underbrace{\left(p - \frac{w}{eS}\right) G''(S)}_{(-)} \underbrace{- \frac{2w}{eS^3} G(S)}_{(-)} + \underbrace{\frac{w}{eS^2} G'(S)}_{(?)}$$

$\Psi_S < 0$ if $G'(S) \leq 0$ unambiguously

$\Rightarrow \frac{\partial S}{\partial e} < 0$

When the SS stock of the resource is initially above the MSY, a productivity increase will lower the SS stock!

The effect becomes ambiguous when $G'(S) < 0$.

Note that as $e \to \infty$, the SS stock becomes $G'(S) = r$. This situation is equivalent to zero cost of harvesting. The opportunity cost of the harvest is determined solely by the stock effect. It is thus not surprising to see that only the natural growth function and the rate of interest matter.

c) $r \to \infty$:

As r becomes arbitrarily large, the only way for the FOC to hold is to have $\left(p - \frac{w}{eS}\right) \to 0$.

$\Rightarrow \lim_{r \to \infty} S^{SS} = \frac{w}{ep}$.

This means that the price of the resource is equal to its marginal cost.

d) Open Access:

With OA, rents are dissipated $\forall t$.

$\Rightarrow pH_t = wE_t$

$\Rightarrow pH_t = w\frac{H_t}{eS_t} \Rightarrow \boxed{p = \frac{w}{eS_t}}, \forall t.$

$\Rightarrow S_t = \frac{w}{ep}, \forall t.$

Note that the SS is reached instantly. Can you explain why?

We have $\lim_{r \to \infty} S^{ss} = S^{OA}$. Can you explain why? Does it have to be always the case that an infinite discount rate be equivalent to open access?

e) Comparisons of SS stocks:

In a), we have that $G'(s) < r$, but it may or may not be negative. Whether S^{SS} is larger than S^{MSY} or not will depend on the shape of the harvesting cost function.

In c) & d), S^{SS} is the same. It will be above the MSY iff $S^{MSY} < \frac{w}{ep}$. If e is large, then it is easy to catch fish and the SS stock will be small, as would be expected.