WEEK 2:

1. Commons and Anti-Commons

(1) Problem of firm i:

$$\max_{x_i} \Pi_i \equiv p x_i \frac{f(x)}{x} - w x_i$$

$$= p x_i (a - bx) - w x_i$$

where $x = \sum_{j=1}^{n} x_j$.

F.O.C. $\frac{\partial \Pi_i}{\partial x_i} = p(a - bx) - pbx_i - w = 0$

$$\Rightarrow pa - pbx_{-i} - 2pbx_i^* = w$$

$$\Rightarrow x_i^* = \frac{pa - pbx_{-i} - w}{2pb}$$

This is the reaction function for each firm i. Assuming symmetry we have:

$$2pbx^* = pa - pb(n-1)x^* - w$$

$$\Rightarrow \boxed{x^* = \frac{pa - w}{(n+1)pb}}$$

The NE total number of boats is then

$$n\alpha^* = \left(\frac{n}{n+1}\right)\left(\frac{pa-w}{pb}\right)$$

The efficient number of boats is found by solving

$$\max_{\alpha} \Pi_{TOT} = pf(\alpha) - w\alpha$$

$$= p(a\alpha - b\alpha^2) - w\alpha$$

F.O.C. $\frac{\partial \Pi_{TOT}}{\partial \alpha} = p(a - 2b\alpha^e) - w = 0$

$$\Rightarrow \alpha^e = \frac{pa - w}{2pb}$$

Hence $n\alpha^* > \alpha^e$ if $n \geq 2$. (overexploitation)

and $n\alpha^* = \alpha^e$ if $n = 1$, as would be expected.

(2) Total Rents:

$$\Pi^*_{TOT} = pf(n\alpha^*) - wn\alpha^*$$

$$= \left(\frac{n}{n+1}\right)\frac{(pa-w)^2}{pb} - \left(\frac{n}{n+1}\right)^2\frac{(pa-w)^2}{pb}$$

$$= \frac{n}{(n+1)^2} \frac{(pa-w)^2}{pb}$$

Maximum is when $n = 1$:

$$\Rightarrow \pi_{TOT}^{MAX} = \left(\frac{1}{4}\right) \frac{(pa-w)^2}{pb}$$

Overexploitation leads to lower rents with $n \geq 2$.

(3) As n increases, $\frac{n}{(n+1)^2}$ decreases. Hence, profits decrease as the number of firms gets larger.

$$\lim_{n\to\infty} \frac{n}{(n+1)^2} = \lim_{n\to\infty} \frac{1}{2(n+1)} = 0$$

(L'Hospital rule).

$n = 1$ implies max. profits since it is equivalent to exclusive ownership.

(4) The unit cost of a boat is now $w + \Sigma q_j$ instead of w.

$$\Rightarrow n x^* = \left(\frac{n}{n+1}\right)\left(\frac{pa - w - \Sigma q_j}{pb}\right)$$

$$\Rightarrow \lim_{n \to \infty} n x^* = \frac{pa - w - \Sigma q_j}{pb}$$

(5) Problem of rights holder j:

$$\max_{q_j} V_j = n x^* \cdot q_j$$

$$= \left(\frac{pa - w - q_j - q_{-j}}{pb}\right) q_j$$

$$\frac{\partial V_j}{\partial q_j} = \frac{pa - w - q_j^* - q_{-j}}{pb} - \frac{q_j^*}{pb} = 0$$

$$\Rightarrow \frac{pa - w - q_{-j}}{2} = q_j^* \text{ : reaction function}$$

By symmetry, we get:

$$q^* = \frac{pa - w - (m-1)q^*}{2}$$

$$\Rightarrow q^* = \frac{pa - w}{m+1} \text{ : Symmetrical NE entry price}$$

Total price is thus: $mq^* = \left(\frac{m}{m+1}\right)(pa-w)$

(6) Equilibrium number of boats:

$$n\alpha^* = \frac{pa - w - \left(\frac{m}{m+1}\right)(pa-w)}{pb}$$

$$= \left(\frac{1}{m+1}\right)\frac{pa-w}{pb}$$

This is less than the efficient number for $m \geq 2$ and a fortiori less than the number found in (1). It is efficient when $n=1$. Too many rights holders with absolute power to exclude leads to an underuse of the fishery.

(7) RENTS:

$$V^*_{TOT} = \left(\frac{1}{m+1}\right)\frac{(pa-w)^2}{pb} - \left(\frac{1}{m+1}\right)^2\frac{(pa-w)^2}{pb}$$

$$= \frac{m}{(m+1)^2}\,\frac{(pa-w)^2}{pb}$$

It is exactly the same expression as the one found in (2) except that m replaces n. We have a symmetric situation between commons and anti-commons.

As m increases, V^*_{TOT} decrease.

$$\lim_{m \to \infty} V^*_{TOT} = 0$$

$$m = 1 \Rightarrow V^*_{TOT} = \frac{(pa - w)^2}{2gb} : \text{efficient.}$$