Common property resources, cooperation, repeated interactions and asymmetric users

a) **Efficiency** the problem is one of choosing individual input levels in order to maximize total profits:

$$\max_{x_A,x_B} \pi_{TOT} = (2-x)x - \frac{1}{2}x_A^2 - \alpha\frac{1}{2}x_B^2. \tag{1}$$

Solving for the first-order conditions yields:

$$x_B^* = \frac{2}{3\alpha+2} \tag{2}$$

$$x_A^* = \frac{2\alpha}{3\alpha+2} \tag{3}$$

The first-order conditions for efficiency imply that the marginal costs are equalized between both users and that the marginal cost is equal to the marginal product.

b) **Free access**

To solve for the Nash equilibrium, we must find each user's reaction function. Noting that $f(x)/x = (2-x)$ For user A, this implies solving:

$$\max_{x_A} \pi_A = (2-x)x_A - \frac{1}{2}x_A^2. \tag{4}$$

Given x_B, this yields the following reaction function:

$$x_A(x_B) = \frac{2-x_B}{3}.$$

And similarly for B, we obtain

$$x_B(x_A) = \frac{2-x_A}{2+\alpha}.$$

We have a Nash equilibrium when each user is on its respective reaction function, which yields:

$$x_A^{FA} = \frac{2(1+\alpha)}{5+3\alpha}, \tag{5}$$

$$x_B^{FA} = \frac{4}{5+3\alpha}. \tag{6}$$

Note that marginal costs are given by $c_A'(x_A) = x_A$ and $c_B'(x_B) = \alpha x_B$. Hence, at the free-access equilibrium, marginal costs are not equalized between users (unless costs are identical, i.e. $\alpha = 1$). This means that we could produce the same output by reallocating some inputs from high marginal cost user to low marginal cost user. Moreover, marginal costs are above marginal products, which means that some inputs are used at a loss.

c) **Repeated interactions with identical users**

With $\alpha = 1$, we have $x_A^{FA} = x_B^{FA} = 0.5$. This yields $\pi_{TOT}^{FA} = 0.75$ and $\pi_A^{FA} = \pi_B^{FA} = 0.375$.

Under efficiency, we have $x_A^* = x_B^* = 0.4$, which yields $\pi_{TOT}^* = 0.8$ and $\pi_A^{FA} = \pi_B^{FA} = 0.4$.

Proposed trigger strategy:

- At any period $t = \tau$, I cooperate fully *if* we have both been cooperating in the past;
- *If not*, revert to the free-access input level.

Using the proposed trigger strategy and assuming that both users have been cooperating in the past by producing at the efficient level yields a present value for user i of

$$V_i^C = \frac{0.4}{1-\beta}.$$

If A considers deviating from the trigger strategy at, say, period t, he will choose the input level that maximizes his gain given $x_B = 0.4$. We have

$$x_A(x_B) = \frac{2-0.4}{3} = 0.5333.$$

Which gives a short term profit of

$$\pi_A^D(t) = (2-(0.4+0.5333))0.5333 - 0.5333^2/2 = 0.4267.$$

Since player b reverts to the free-access output level thereafter, A's profit in future periods is 0.375. Hence, the present value of deviating from the trigger strategy is

$$V_A^D = 0.4267 + \beta 0.375 + \beta^2 0.375 + \beta^3 0.375 + ... \tag{7}$$

$$= 0.4267 + \beta \frac{0.375}{\beta}. \tag{8}$$

User A will stick to the trigger strategy iff $V_A^C > V_A^D$, which implies $\beta > 0.5168$.

d) **Repeated interactions with asymmetric users**

With $\alpha = 2$, efficiency dictates $x_A^* = 0.5$, $x_B^* = 0.25$ and $x = 0.75$. This implies $\pi_A^* = 0.5$ and $\pi_B^* = 0.25$.

With free access, we have $x_A^{FA} = 6/11$, $x_B^{FA} = 4/11$ and $x^{FA} = 10/11$. This yields $\pi_A^{FA} = 0.4463 < \pi_A^*$ and $\pi_B^{FA} = 0.2645 > \pi_B^*$.

Since $\pi_B^{FA} > \pi_B^*$, it would be impossible to sustain cooperation at the efficient output level even with infinitely repeated interactions while using the proposed trigger strategy. This result suggests that asymmetric cost structures make cooperation more difficult to sustain. User A would have to compensate user B in order to induce cooperation and efficient use.