Non-renewable resource exploitation, stock discovery and anticipations

We wish to compare how a similar increase in the stock of a non-renewable resource can affect its price and extraction paths when it is anticipated versus when it is non-anticipated. Use a four-quadrant graph to analyze this (See accompanying graphic file.)

a) In the first case, assume that you are now at time $t = 0$ and that the stock size is S_0. The stock is anticipated to increase by amount S_1 at a future specific date, say at date $t = t_0 > 0$. Assume that $S_0 + S_1 = \bar{S}$. *(NB Depending on the relative sizes of S_0 and S_1, there are two cases to consider.)*

b) In the second case, you are now at date $t = 0$ and the change occurs at that same future time $t = t_0 > 0$, but it is a total surprise.

c) Compare the two cases and interpret.

LET THE INITIAL RESOURCE STOCK $S(0)$ BE EQUAL TO S_0. THEN, AS LONG AS $S_t > 0$, THE OPTIMAL EXTRACTION PATH MUST RESPECT HOTELLING'S RULE

$$\frac{\dot{P}}{P} = r$$

AND THE RESOURCE CONSTRAINT

$$\int_0^T R_t dt = S_0.$$

THIS PATH IS REPRESENTED BY CURVE $P_0 - A$ IN THE ACCOMPANYING GRAPHIC SOL-GRAPH-STOCK-INCREASE-ANTICIPATION.PDF.

(A) IT IS NOW FULLY ANTICIPATED THAT THE STOCK WILL INCREASE BY SOME AMOUNT S_1 AT SOME FUTURE TIME $t = t_0$. LET $S_0 + S_1 = \bar{S}$. PROVIDED THAT

(1)
$$\int_0^{t_0} R'_t dt \leq S_0,$$

PRICE PATH $P'_0 - B$ MUST BE OPTIMAL IF IT RESPECTS HOTELLING'S RULE AND IF

$$\int_0^{T'} R'_t dt = \bar{S}.$$

IT MUST BE THE OPTIMAL PATH BECAUSE WE WOULD ALSO CHOOSE THAT PATH IF WE HAD $S(0) = \bar{S}$ TO START WITH. HENCE, AS LONG AS CONSTRAINT (1) IS NOT BINDING, WE WOULD CHOOSE THAT SAME PRICE PATH.

BUT WHAT IF CONSTRAINT (1) IS BINDING, I.E. ALONG PRICE PATH $P'_0 - B$, WE HAVE

(2)
$$\int_0^{t_0} R'_t dt > S_0?$$

Then the initial stock becomes exhausted before $t = t_0$ and the price jumps to the choke price k until the new stock becomes available at t_0. This cannot be optimal since the jump in price provides an infinite return on the resource. Hence, the price must be increased at $t = 0$ in such a way that the initial stock becomes exactly depleted at $t = t_0$. This implies that $P''_{t_0} < P'_{t_0}$ since, provided $S_0 + S_1 = \bar{S}$ in both cases,

$$\int_0^{t_0} R''_t dt < \int_0^{t_0} R'_t dt \Rightarrow \int_{t_0}^{T''} R''_t dt > \int_{t_0}^{T'} R'_t dt. \tag{3}$$

This yields price path $P''_0 - C - D - E$ when constraint (2) holds.

(b) When the increase in stock is <u>not anticipated at all</u>, then the resource is initially exploited as if the total stock were S_0. The initial price is $P(0) = P_0$ and the price follows price path $P_0 - A$ up to $t = t_0$. At t_0, the sudden increase in stock leads to a drop in price, lest some resources will be left in the ground once the choke price is reached. Note that the price will drop to a lower value than P''_{t_0} since some of the initial reserves are still available at t_0, i.e. $S'''(t_0) > S''(t_0)$. The resulting price path is represented by $P_0 - F - G - H$.