A dynamic, discrete-time analysis of a fishery with ecological stock value

a) Problem of the social planner:

$$\max_{\{H_t, S_t\}} V = \sum_{t=0}^{\infty} \beta^t \left\{ U(S_t) + p H_t - C(H_t, S_t) \right\}$$

$$\text{s.t. } S_{t+1} - S_t = F(S_t) - H_t$$

Substituting for $H_t = F(S_t) + S_t - S_{t+1}$:

$$\frac{\partial V}{\partial S_t} = \beta^{t-1}\left[-p + C_H(H_{t-1}, S_{t-1})\right]$$
$$+\beta^t\left[U'(S_t) + p(F'(S_t)+1) - C_H(H_t, S_t)(F'(S_t)+1)\right.$$
$$\left. - C_S(H_t, S_t)\right] = 0$$

This is the no-arbitrage condition between two consecutive periods. It says that along the optimal path, one must be indifferent between leaving one more unit at $t-1$ and extracting it at t, or not extracting it at t.

Reducing harvest by one marginal unit at $t-1$ has a net cost of $C_{H,t-1} - p$ in terms of forgone income.

The benefit at period t comes in terms of higher indirect stock benefits $U'(S_t)$, higher harvesting $(p - C_{H,t})(1 + F'(S_t))$, and lower harvest cost $C_{S,t}$.

$$\Rightarrow \frac{(p-C_{H,t})-(p-C_{H,t-1})+U'(S_t)-C_{S,t}+(p-C_{H,t})F'(S_t)}{p-C_{H,t-1}} = r \quad (2)$$

b) <u>Steady - State</u>: (We drop the t subscript.)

$$H = F(S).$$

$$\Rightarrow -(p-C_H)+B\{U'(S)+(p-C_H)(1+F'(S))-C_S\}=0$$

where $B = \frac{1}{1+r}$.

$$\Rightarrow (1+r)(p-C_H) = U'(S) + \cancel{p-C_H} + (p-C_H)F'(S) - C_S$$

$$\Rightarrow \frac{U'(S)+(p-C_H)F'(S)-C_S}{p-C_H} = r$$

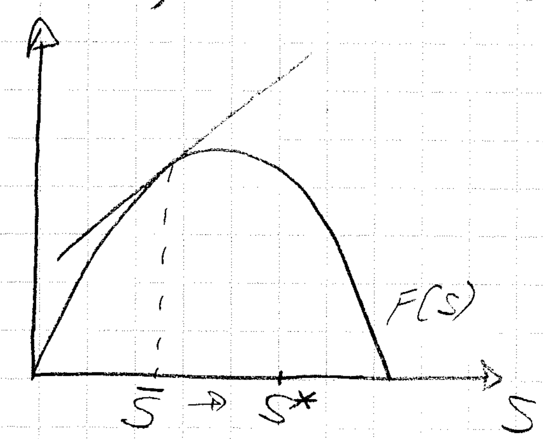
$$\Rightarrow F'(S^*) = r - \frac{U'(S^*)-C_S}{p-C_H}$$

Assume $p > C_H^{ss}$. Then $F'(S) < r$ in S-S.

Define $\bar{S}$ as $F'(\bar{S}) = r$,

then $S^{ss} > \bar{S}$.

We could have $F'(S) < 0$.



The combined stock effects on harvesting costs and indirect social value lead to a larger stock level in the S-S equilibrium.

c) Suppose that technological improvement leads to $C = 0$.

$\Rightarrow C_H = C_S = 0$.

$$\Rightarrow F'(\hat{S}) = r - \frac{U'(\hat{S})}{p}$$

Since $p > p - C_H$ and $U'(S) < U'(S) - C_S, \forall S$, we have $-\frac{U'(S^*)}{p} > -\frac{U'(S^*) - C_S(H^*, S^*)}{p - C_H(H^*, S^*)}$.

Hence, $\hat{S} < S^*$. The S-S stock will be smaller with the technological improvement. Note that there are two reasons for this. One is that the marginal harvest cost is lower, which increases the marginal rent value $(p - C_H)$ and thus reduces the relative return from stock dividends $\left(\frac{U'(S) - C_S}{p - C_H}\right)$. The other is that the dividends from leaving a larger stock is now lower in absolute terms by value C_S.

d) A profit-max. firm would usually not account for indirect value $U(S_t)$. Hence, its S-S choice is determined by S^F such that:

$$F'(S^F) = r + \frac{C_S}{p - C_H} > r - \frac{U'(S^F) - C_S}{p - C_H}$$

Hence, $S^F < S^*$. As would be expected, the firm has lower incentives to conserve the resource as it does not account for social dividends $U'(S)$.

Q: Can you think of a way to re-establish optimality?

e) We cannot tell "a priori" if $F'(S^*) \gtrless 0$, i.e. $S^* \lessgtr S^{MSY}$. If r is very large, then it is more likely that $F'(S^*) > 0$ and thus $S^* < S^{MSY}$. But the opposite holds if $U'(S)$ or $-C_S$ are large, i.e. resource conservation brings large dividends in terms of indirect social benefits or lower harvest costs.