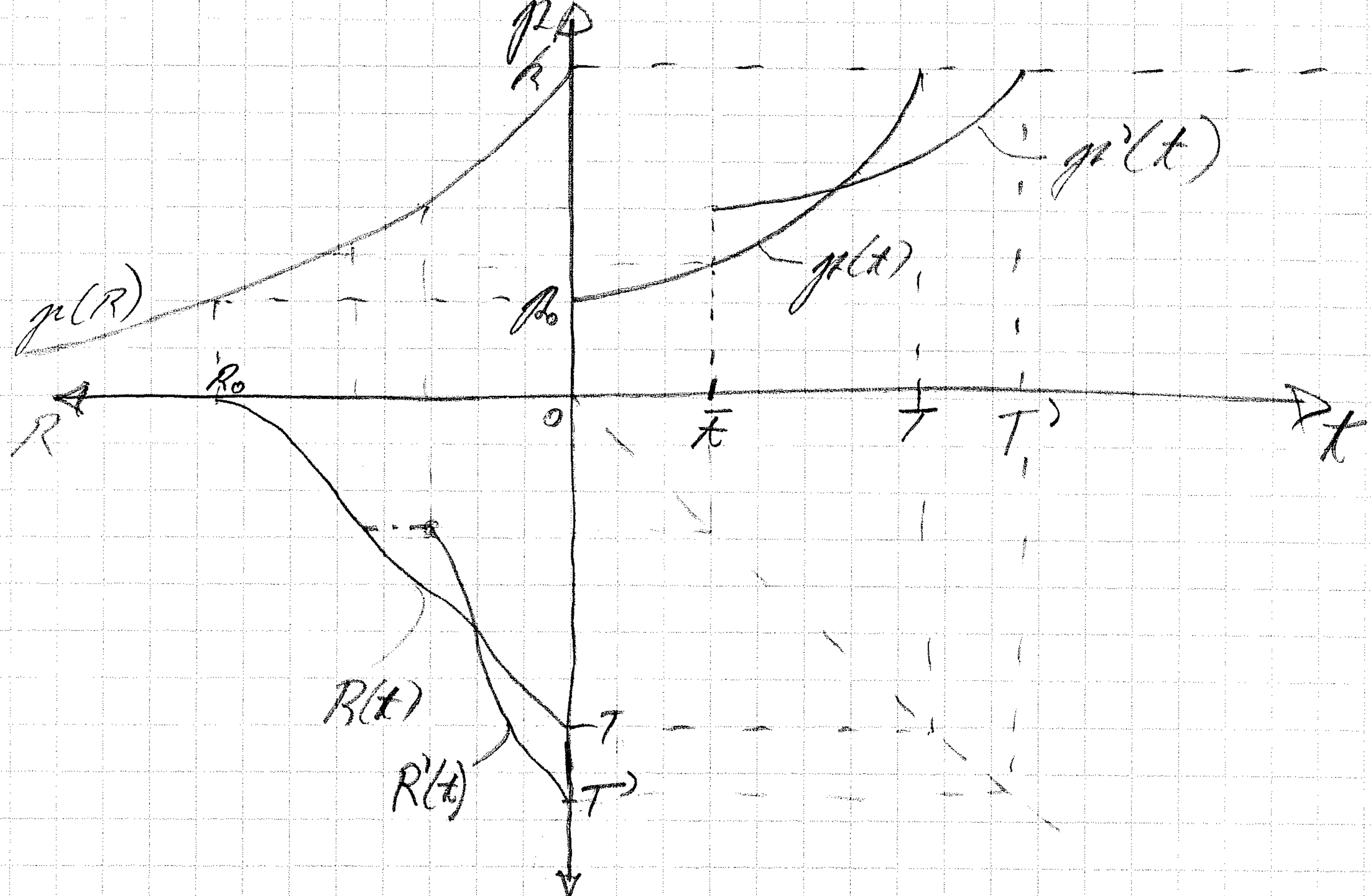
ECO 6143
FINAL EXAM 2008

1) Financial crisis & oil prices

Let us use the four-quadrant graphic to analyse the effect of an unexpected drop in asset returns



$k \equiv$ choke price
$p(R) \equiv$ demand per period (inverse)
$R \equiv$ extraction rate
$S_0 \equiv$ initial stock size
} given parameters of problem

Given the above, if the interest rate is equal to some value r_0 the extraction rate must respect the following conditions:

$\frac{\dot{p}}{p} = r_0$: Hotelling rule

$\int_0^T R(t)\,dt = S_0$: No oil left in the ground

$p(T) = k$: No jump in price at the end!

Assume that the above three conditions are respected at interest rate r_0.

Suppose that the interest rate unexpectedly drops to $r_1 < r_0$ at time $\bar{t}$. For $t > \bar{t}$, the price path must respect:

$$\frac{\dot{p}}{p} = r_1 < r_0$$

The new price path increases at a slower rate. This leads to a jump in the price at $\bar{t}$. Otherwise, the future price would be everywhere lower, leading to resource extraction before the choke price is reached. This is illustrated by price path $p'(t)$.

We have thus shown that a drop in asset returns leads extractors to increase the price of oil by reducing the extraction rate. The reverse is also true, i.e. a jump in asset returns will lead extractors to reduce prices by extracting more. This may explain oil price movements since 1998 as follows:

① The gradual drop in asset returns in the USA from 1998 to 2007 coincides with a gradual increase in oil prices.

② The subprime crisis in the summer of 2007 led many investors to suddenly expect a large drop in asset returns. This corresponds to a large increase in oil prices in a short period.

③ By the summer of 2008, asset prices were so low that many investors considered them a bargain, i.e. expected returns were suddenly high again. Our analysis predicts that this should lead to a drop in oil prices as supplies go up, as actually happened.

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#2) a) <u>Efficient use:</u>

$$\max_{Z} \Pi = p f(Z) - c Z$$

$$\frac{\partial \Pi}{\partial Z} = p f'(Z) - c = 0$$

$$\Rightarrow p f'(Z^*) = c$$

Z^* = efficient no of cows.

b) <u>Free-access:</u>

The problem of rancher i is:

$$\max_{Z_i} \Pi_i = Z_i p \frac{f(Z)}{Z} - c Z_i \quad \text{where } Z = \sum_{j=1}^{n} Z_j .$$

$$\frac{\partial \Pi_i}{\partial Z_i} = p \frac{f(Z)}{Z} + Z_i\, p \frac{\partial}{\partial Z}\left(\frac{f(Z)}{Z}\right) - c = 0$$

In the symmetric NE, $Z_1 = Z_2 = \dots = Z_n = Z^e$.
Let $\bar{Z} = n Z^e$. We have

$$p \frac{f(\bar{Z})}{\bar{Z}} + \frac{\bar{Z}}{n}\, p \frac{\partial}{\partial Z}\left(\frac{f(\bar{Z})}{\bar{Z}}\right) = c$$

This characterizes the NE with n ranchers. When $n \to \infty$, we have

$$p \frac{f(\bar{Z})}{\bar{Z}} = c$$

$\Rightarrow$ All rents are exhausted.

c) Cattle thieves:

$g(z) \equiv$ number of cows being stolen, with $g'(z) > 0$.

The problem of one owner is:

$$\max_{z} \Pi = p f(z) - c z - p g(z)$$

$$\frac{\partial \Pi}{\partial z} = p f'(z^t) - c - p g'(z^t) = 0$$

$$\Rightarrow p f'(z^t) = c + p g'(z^t) > c.$$

Since $p f'(z^t) > c$, we have

$$\boxed{z^t < z^* < \bar{z}}$$

Cattle theft leads to the pasture being underused while free access to the pasture leads to overuse.

As far as the state of the resource stock is concerned, imperfect property rights may lead to opposite outcomes, depending on the form that they take.