1. STEADY-STATE ANALYSIS OF A FISHERY

1) The S-S is characterized by

$$H^{SS} = G(S^{SS}).$$

NB We drop the t & SS subscripts for clarity of exposition.

$$\Rightarrow eES = g\left(1 - \frac{S}{\bar{S}}\right)S$$

$$\Rightarrow E = \frac{g}{e}\left(1 - \frac{S}{\bar{S}}\right)$$

$$\Rightarrow S^{SS} = \bar{S}\left(1 - \frac{e}{g}E\right)$$

This represents the steady-state stock w.r.t. any effort level. Note that the stock is depleted if $E \geq g/e$.

$$\Rightarrow H^{SS} = eES^{SS} = eE\bar{S}\left(1 - \frac{e}{g}E\right)$$

$$= e\bar{S}E - \frac{e^2}{g}\bar{S}E^2$$

$$= aE - bE^2 = (a - bE)E$$

where $a = e\bar{S}$

$$b = \frac{e^2}{g}\bar{S}$$

Note that we have a quadratic output function. The S-S harvest rate is nil at $E=0$ and $E = a/b$.

The MSY is given by

$$\frac{\partial H^{ss}}{\partial E} = 0 \Rightarrow a - 2bE_{MSY} = 0$$

$$\Rightarrow E_{MSY} = \frac{a}{2b} = \frac{e\bar{S}}{2e^2\frac{\bar{S}}{g}} = \boxed{\frac{g}{2e}}$$

$$\Rightarrow S_{MSY} = \bar{S}/2$$

$$\Rightarrow H_{MSY} = e\bar{S}\frac{g}{2e}\left(1 - \frac{e}{g}\frac{g}{2e}\right) = \boxed{\frac{g\bar{S}}{4}}$$

N.B. It is also possible to find through maximization of the $G(S)$ function. Good way to verify calculations up to now.

2) OPEN-ACCESS FISHERY

$\Rightarrow$ NO RENTS.

$\Rightarrow TR = TC$

$\Rightarrow pH^{ss}(E) - wE = 0$

$\Rightarrow p(a - bE)\cancel{E} - w\cancel{E} = 0$

$$\boxed{E_{OA} = \frac{g}{e}\left[1 - \frac{1}{e\bar{S}}\frac{w}{p}\right]}$$

$$\Rightarrow S^{OA} = \bar{S}\left(1 - \frac{e}{g}\cdot\frac{g}{e}\left(1 - \frac{1}{e\bar{S}}\frac{w}{p}\right)\right)$$

$$\boxed{S^{OA} = \frac{1}{e}\,\frac{w}{p}}$$

$$\Rightarrow \boxed{H^{OA} = g\left(1 - \frac{w}{ep\bar{S}}\right)\frac{w}{ep}}$$

3) From the above results, we have:

$\Delta^+ p \Rightarrow \Delta^- S^{OA}$, $\Delta^+ E^{OA}$

ΔH^{OA} is ambiguous. It depends on where we are w.r.t. the MSY.

$\Delta^- w \Rightarrow$ Same effect as $\Delta^+ p$ since it is really the ratio $\frac{w}{p}$ that counts.

$\Delta^+ e \Rightarrow \Delta^- S^{OA}$

ΔE^{OA} and ΔH^{OA} are ambiguous.

4) EXCLUSIVE PROPERTY (EP):

Rent maximization implies:

$$MR = MC$$

or $$\frac{\partial}{\partial E}\left[p H^{ss}(E) - w E\right] = 0$$

$$\Rightarrow p(a - 2bE) - w = 0.$$

$$\Rightarrow E_{EP} = \frac{a}{2b} - \frac{w}{2bp} = \frac{1}{2} E^{OA}$$

$$\boxed{E_{EP} = \frac{g}{2e}\left[1 - \frac{1}{e\bar{S}}\frac{w}{p}\right]}$$

$$\Rightarrow \boxed{S_{EP} = \frac{\bar{S}}{2} + \frac{1}{2e}\frac{w}{p}}$$

$$\boxed{H_{EP} = \frac{g}{4}\left(\bar{S} - \frac{1}{\bar{S}}\left(\frac{w}{ep}\right)^2\right)}$$

5) $\left.\begin{matrix}\Delta^+ p \\ \Delta^- w\end{matrix}\right\} \Rightarrow \Delta^+ E_{EP},\ \Delta^- S_{EP},\ \Delta^+ H_{EP}$

$\Delta^+ e \Rightarrow \Delta^- S_{EP},\ \Delta^+ H_{EP}$

ΔE_{EP} is ambiguous.

N.B. Answer to 6) on typed PDF file.