Non-renewable resource extraction
when costs depend on remaining stock levels

A firm owns a mineral reserve and must decide on how fast to extract the mineral in order to maximize the present value of this asset. Here are some parameters of the problem:

r is the firm's discount rate (i.e. the return from the best alternative investment);
$p(t)$ is the unit selling price of the mineral, with $0 \leq \dot{p}(t) < \infty$ (the firm is a price-taker);
$S(t)$ is the stock of mineral left at time t, with S_0 as the initial stock;
$c(S(t))$ is the *unit* extraction cost, with $c'(S(t)) < 0$, $c''(S(t)) > 0$ and $c(0) = \bar{c} \leq \infty$;
T is the final extraction time, i.e. the time horizon goes from $t = 0$ to $t = T$, $T \leq \infty$.

The analysis is performed in continuous time and we denote the instantaneous extraction rate by $q(t) \geq 0$. (*NB All your answers must include some intuitive interpretations.*)

(1) Find the intertemporal extraction arbitrage condition and interpret it.
(2) Analyse the case with $\dot{p}(t) = 0$. What does this suggest about the role an increasing resource price in the present setting? (Suggestion: Draw a diagram of the unit cost curve $c(S)$ to compare it with the resource price.)
(3) Consider now the case with $\dot{p}(t) > 0$. Assume, for simplicity, that $p(t) = p_0 \exp^{\gamma t}$, such that $\dot{p}/p = \gamma$, where $0 < \gamma < r$.
   (a) Give conditions under which the extraction rate will be infinite at $t = 0$.
   (b) Conversely, give conditions under which the extraction rate will be nil for some initial interval $t \in (0, t')$, but will become positive after t'. Characterize t'.
   (c) Analyse the case with $\gamma \geq r$.