EXERCISES AND PROBLEMS
WEEK 3 (Winter 2007)

1. Steady-State Analysis of a Fishery

The natural continuous-time rate of change of a fish stock is given by

$$G(S_t) = g\left(1 - \frac{S_t}{\bar{S}}\right) S_t,$$

where S_t, $\dot{S}_t$, g and $\bar{S}$ respectively denote the time t stock of fish in tons, instantaneous rate of change of the stock, intrinsic growth rate of the stock and maximum natural stock size. Assume that the rate of harvest H_t is given by

$$H_t = eE_tS_t,$$

where E_t is the total fishing effort and e is a productivity parameter.

(1) Derive analytically the steady-state yield-effort curve for this fishery. Determine the maximum sustainable yield and its corresponding effort level.
(2) Assume a constant unit selling price of a ton of fish of p and a constant unit cost of fishing effort of w. Determine analytically the equilibrium level of harvest, effort and stock size assuming open access to the fishery.
(3) Analyze the effects of higher p, lower w and higher productivity e on the steady-state values.
(4) Solve for question 2 assuming exclusive property.
(5) Answer question 3 assuming exclusive property.
(6) Compare your answers to 3 and 5 and discuss.

2. Analysis of a fishery with a demand curve (From Hartwick and Olewiler, 1998, p 135)

Suppose that the demand curve for flounder is given by $P = 400 - 3H$, where P is the price of flounder and H is the harvest in thousand of pounds. The *sustainable catch* is given by $H = aE - bE^2 = 0.6E - 0.0015E^2$, where E is the level of effort. Unit costs are assumed constant and calculated to be about \$200 per unit of fishing effort. Given this information compute *graphically*:

(1) The open access harvest and level of effort
(2) The private property harvest and level of effort, assuming that a competitive firm owns the fishery, i.e. it takes the price as given.
(3) The private property harvest and level of effort, assuming that a monopolist owns the fishery.

3. Grazing land as a renewable resource (discrete time analysis)

The production function for *annual* beef production on a pasture is given by

$$B_t = G_t B(H_t), \quad \text{with } B_H > 0, \; B(0) = 0, \; \lim_{H_t \to \infty} B(H_t) = 1$$

where B_t, G_t, and H_t respectively denote the beef produced (tons), the amount of grazable grass (tons), and the number of beef cattle (heads) allowed to graze. The amount of grass available in year $t+1$ depends solely on the number of beef cattle used the preceding year, i.e.

$$G_{t+1} = G(H_t),$$

with $G'(H_t) < 0$, $G(0) = G_0$, $\lim_{H_t \to \infty} G(H_t) = 0$. Beef sells at a price of p per ton, and the cost of herding, transporting, and processing *each* head of cattle is c.

(1) Given G_t, *characterize* the conditions for
  (a) the maximum beef production.
  (b) the number of cattle that will maximize year t's profits.
  (c) The *open access* number of cattle.

  Interpret briefly your results. (We assume that the second-order conditions for a maximum are always satisfied.)

(2) Characterize the conditions for a steady-state equilibrium herd size under open access, year-to-year profit maximization, and maximum sustainable yield. (NB Since this is a discrete-time problem, a steady state implies that $G_{t+1} = G_t$.)