EXERCISES AND PROBLEMS
WEEK 2 (Winter 2007)

1. Free access and regulation (Based on DH (1979).)

We wish to further analyze the problem of free access to a fishery with n identical fishers seen in class. We consider still the symmetrical Nash equilibrium. (NB Provide an intuitive explanation for each answer.)

(1) Find the *tax rate* per unit of effort that would re-establish optimality. What does it represents exactly?
(2) How does the unit tax value vary with the number of firms?
(3) Do fishing firms prefer the free access regime with the tax or without? Analyze as completely as possible.
(4) Assume instead that the government chooses to distribute licenses to fishing firms. What will be the equilibrium price of each license?
(5) Do firms prefer the license to the tax?
(6) Discuss whether those two regulation instruments really change anything fundamental to the initial problem of free access.

2. Commons and anti-commons (Based on Dasgupta and Heal, 1979 and Buchanan and Yoon, 2000)

Assume that the total harvest function of a fishery is quadratic, i.e.

$$f(x) = ax - bx^2$$

where x denotes the total number of identical boats operating. There are n fishing firms, $i = 1, ..., n$, that can *freely* access the fishery at a cost of w per boat. Boats are the only input. Each harvest unit fetches a constant price p at the market.

(1) Determine the symmetrical Nash equilibrium number of boats per firm operating on the fishery. How does the total number of operating boats compare to the efficient level?
(2) Calculate the total rents that the fishery generates with n fishing firms and compare to the maximum that could be attained. Comment.
(3) Comment on what happens to the total number of boats and total rents when n increases, when $n = 1$ and when $n \to \infty$.

Assume now that there are m absolute rights holders to the same fishery, $j = 1, ..., m$. In order to send a boat on the fishery, a firm must ask permission to each one of those rights holders who will then demand a compensation. For each operating boat, let us say that rights holder j asks a price q_j, with $0 \leq q_j < \infty$. Hence, a firm must pay a total of $\sum_{j=1}^{m} q_j$ to operate a boat, on top of the standard cost w.

(4) Determine the total number of boats operating on the fishery *for given* $\sum_{j=1}^{m} q_j$.

For questions (5) to (7), assume that $n \to \infty$.

(5) Determine the symmetrical Nash equilibrium entry price q_j asked by each rights holder.
(6) How does the total number of operating boats compare to the efficient one and to the number found in (1)? Comment.
(7) What happens to rents when m increases, when $m = 1$ and when $m \to \infty$. Compare with (3) and comment.

(8) `Ph.D. students (Optional for MA students.)` Try analyzing the problem assuming finite n. To fix ideas, you may want to start with $n = 2$. Give some intuition as to what is going on.

3. Private ownership with costly exclusion

As discussed briefly in class, one should make a difference between the right to exclude and the real ability to do so. This exercise considers the case of a single owner of a resource who decides on how to exploit the resource but must pay to exclude potential trespassers, referred to as poachers here.

Poaching can be viewed as a *sequential* game between a resource owner and n poachers. In the first stage, the owner decides on the number of hours of labor he will hire ($L \geq 0$) to exploit the resource, say a fishery, and on the intensity with which he monitors poaching. As a result of this policing, each poacher expects to be caught with probability $\lambda \in (0, 1)$. If the owner catches a poacher, he confiscates his catch but can exact no other penalty.

We initially *restrict attention* to the second stage where the n poachers choose the number of hours of illegal activity simultaneously, after *observing* both L and λ. Assume that each poacher wishes to maximize his expected gain. Each poacher has T hours per day to work and can divide them between legal work and poaching. Legal work pays w per hour and the stolen catch sells for p per unit. If player i poaches for h_i hours, he earns in expectation:

$$\lambda w(T - h_i) + (1 - \lambda) \left(w(T - h_i) + \frac{h_i}{h_i + h_{-i} + L} pF(h_i + h_{-i} + L) \right),$$

where $F(\cdot)$ is the total output function, $F'(\cdot) > 0$, $F''(\cdot) < 0$, and $h_{-i} = \sum_{j \neq i} h_j$.

(1) Find the symmetric Nash equilibrium conditions for any given pair (L, λ). (Consider only the interior conditions, i.e. $h_i^* < T$.)

(2) Assume now that poaching is *organized* by a criminal gang that controls the number of poachers in order to maximize their total expected profits. Characterize the equilibrium condition for h_i, $i = 1, ..., n$, in this case. Compare with your result in (1) and comment.

For the rest of this question, we assume unorganized poaching, as in (1).

(3) Denote the aggregate poaching hours as $H(L, \lambda) = \sum_{i=1}^{n} h_i(L, \lambda)$. Verify that H is strictly decreasing in L and λ for $H > 0$.

(4) For fixed L and λ, characterize the free-access limit case where $n \to \infty$. What is the equilibrium value of the average product of the resource? Interpret.

(5) `Ph.D. students (Optional for MA students.)` Let us now turn to the owner's problem while assuming that n is very large, i.e. the case where $n \to \infty$. For fixed policing λ, derive the owner's first-order conditions for L (don't forget that he is a first-mover). What is the equilibrium poaching level H induced by the owner's choice? Interpret.