PROBLEM SET 7 (Fall 2008)

1. Common property resources, cooperation, repeated interactions and asymmetric users

A common-property resource is accessed by two users A and B. The total output is given by quadratic output function

$$f(x) = (2 - x)x,$$

where x denotes the sum of individual input effort, i.e. $x = x_A + x_B$. The users may differ by the cost of their effort. The respective total costs are given by

$$\begin{aligned} c_A(x_A) &= \frac{1}{2}x_A^2, \\ c_B(x_B) &= \alpha\frac{1}{2}x_B^2, \text{ with } \alpha > 0. \end{aligned}$$

Those costs are given in units of the resource.

a) **Efficiency** Give the conditions that characterize the efficient allocation of efforts x_A^* and x_B^* between the two users. Provide a brief economic interpretation.

b) **Free access** Derive the conditions for the non-cooperative Nash equilibrium individual level of effort x_A^{FA} and x_B^{FA} assuming a free access regime. Assume that each user's average product of effort is equal to the global average product of effort $f(x)/x$. Compare with the efficient allocation conditions found in (a) and interpret briefly.

c) **Repeated interactions** Suppose now that both users have the same cost structures, i.e. $\alpha = 1$. Calculate total and individual profits in both (a) and (b). Suppose that the CPR is still subject to free access as in (b) but that instead of meeting just once, the game in (b) is just *one stage* in an infinitely repeated game. Assume that each user has a discount factor between period equal to $\beta \in (0, 1)$. Can you propose a *trigger strategy* that each user could adopt in order to re-establish efficiency on the CPR? Explain and interpret.

d) **Asymmetric users** Suppose now that user B has higher costs than user A, say with $\alpha = 2$. Calculate total and individual profits in both (a) and (b). Can you propose a *trigger strategy* that each user can adopt in order to re-establish efficiency on the CPR? What does this say about cooperation on CPR between asymmetric users? Explain and interpret.

2. Population size, conflict and sustainable resource use

When a new track of land is being settled at some remote location, settlers have a choice between a sustainable use of the land or land mining. A sustainable use produces a constant flow of output y while mining produces an instantaneous gain of S. In both cases, the unit price of the output is equal to 1. Given an interest rate of r, we assume that a sustainable use of the land is *a priori* preferable with $y/r > S$.

The problem is that if the first settler to arrive decides for a sustainable use of the land, he must also protect it from other claimants. We assume that there are n claimants, including the first settler. If claimant i expends effort level x_i to appropriate the track of land, he has a probability

$$\frac{x_i}{\sum_{j=1}^{n} x_j}$$

of becoming the owner, in which case he benefits from the sustainable use of the land forever. Assuming that the unit cost of effort is c for all claimants, the *expected* value of the contest for a sustainable use for claimant i is thus

$$V_i = \frac{y}{r} \frac{x_i}{\sum_{j=1}^{n} x_j} - cx_i. \tag{1}$$

a) Assume for now that the first settler decides for a sustainable use of the land. He thus enters into a contest with $n-1$ other claimants. Derive the symmetrical Nash equilibrium level of effort x_i that will be expended by each contestant as a function of y, r, c and n.
b) Calculate the equilibrium value V_i^* for the first contestant of a sustainable use of the land.
c) Suppose that n is a measure of a country's population size. Compare V_i^* with S and argue that as the population size increases, it becomes less likely that settlers will opt for a sustainable use of land in new settlements.

3. Private ownership with costly exclusion

One should make a difference between a de jure *right to exclude and a* de facto *ability to do so. This exercise considers the case of a single owner of a resource who decides on how to exploit the resource but must pay to exclude potential trespassers, referred to as poachers here.*

Poaching can be viewed as a *sequential* game between a resource owner and n poachers. In the first stage, the owner decides on the number of hours of labor he will hire ($L \geq 0$) to exploit the resource, say a fishery, and on the intensity with which he monitors poaching. As a result of this policing, each poacher expects to be caught with probability $\lambda \in (0, 1)$. If the owner catches a poacher, he confiscates his catch but can exact no other penalty.

We initially *restrict attention* to the second stage where the n poachers choose the number of hours of illegal activity simultaneously, after *observing* both L and λ. Assume that each poacher wishes to maximize his expected gain. Each poacher has T hours per day to work and can allocate them between legal work and poaching. Legal work pays w per hour and the stolen catch sells for p per unit. If player i poaches for h_i hours, he earns in expectation:

$$\lambda w(T - h_i) + (1 - \lambda) \left(w(T - h_i) + \frac{h_i}{h_i + h_{-i} + L} pF(h_i + h_{-i} + L) \right),$$

where $F(\cdot)$ is the total output function, $F'(\cdot) > 0$, $F''(\cdot) < 0$, and $h_{-i} = \sum_{j \neq i} h_j$.

(1) Find the symmetric Nash equilibrium conditions for any given pair (L, λ). (Consider only the interior conditions, i.e. $h_i^* < T$.)

(2) Assume now that poaching is *organized* by a criminal gang that controls the number of poachers in order to maximize their total expected profits. Characterize the equilibrium condition for h_i, $i = 1, ..., n$, in this case. Compare with your result in (1) and comment.

For the rest of this question, we assume unorganized poaching, as in (1).

(3) Denote the aggregate poaching hours as $H(L, \lambda) = \sum_{i=1}^{n} h_i(L, \lambda)$. Verify that H is strictly decreasing in L and λ for $H > 0$.
(4) For fixed L and λ, characterize the free-access limit case where $n \to \infty$. What is the equilibrium value of the average product of the resource? Interpret.
(5) Let us now turn to the owner's problem while assuming that $n \to \infty$. For fixed policing λ, derive the owner's first-order conditions for L (don't forget that he is a first-mover). What is the equilibrium poaching level H induced by the owner's choice? Interpret.