## PROBLEM SET 7 (Fall 2007)

### 1. A common property resource with heterogeneous users

A common-property resource is accessed by two users $A$ and $B$. The total output is given by quadratic output function

$$f(x) = (2 - x)x,$$

where $x$ denote the sum of individual input effort, i.e. $x = x_A + x_B$. The users differ by the cost of their effort. The respective total costs are given by

$$\begin{aligned} c_A(x_A) &= \frac{1}{2}x_A^2, \\ c_B(x_B) &= x_B^2. \end{aligned}$$

(a) Find the efficient allocation of effort $x_A^*$ and $x_B^*$ between the two users. Provide a brief economic interpretation. Calculate the total profit level.

(b) Find the (non-cooperative) Nash equilibrium individual level of effort $x_A^{FA}$ and $x_B^{FA}$ assuming a free access regime. Assume that each user's average product of effort is equal to the global average product of effort $f(x)/x$. Calculate the individual profit levels. Compare with the efficient allocation found in (a) and interpret briefly.

(c) Suppose that the users get together in order to assign *non-transferable* quotas on each other's effort level equal to the efficient level, i.e. $q_A^{NT} = x_A^*$ and $q_B^{NT} = x_B^*$. Participation is purely voluntary *ex-ante*. But once it is agreed upon, each user strictly adheres to its quota level, i.e. there is no enforcement problem. Using the free access Nash equilibrium as the benchmark, show that user B will not agree to participate in this scheme. (NB This is essentially equivalent to a non-transferable quota scheme without subsidy.)

(d) Suppose now that quotas are transferable. They are initially distributed in the same proportion as the proportion of individual effort that occurs in the non-cooperative free access equilibrium derived in (b), i.e.

$$\begin{aligned} q_A^T &= \frac{x_A^{FA}}{x^{FA}}x^*, \\ q_B^T &= \frac{x_B^{FA}}{x^{FA}}x^*, \end{aligned}$$

where $x^{FA} = x_A^{FA} + x_B^{FA}$ and $x^* = x_A^* + x_B^*$. (This is similar to a *grandfather clause* in which the worst offender actually gets a higher share of quotas.)

i) Show that there are gains from trade such that user A buys $q_B^T - x_B^*$ units of effort quotas from B, thus leading to an efficient allocation of effort. (Hint: You must show that the increase in profit for user A are larger than the drop in profits for user B. Hence there exists a price range for which both will gain from trading $q_B^T - x_B^*$ quotas.)

ii) Show that both users will choose to participate in this scheme once we account for the transfers due to the quotas' price level.

(e) The above results can be generalized to common property resource users with heterogeneous characteristics. Discuss the consequences for the possibility of reaching a CPR sharing agreement.

## 2. Population size, conflict and sustainable resource use

When a new track of land is being settled at some remote location, settlers have a choice between a sustainable use of the land or land mining. A sustainable use produces a constant flow of output $y$ while mining produces an instantaneous gain of $S$. In both cases, the unit price of the output is equal to $p(d)$, where $d$ is distance to market. Given an interest rate of $r$, we assume that a sustainable use of the land is *a priori* preferable with $p(d)y/r > p(d)S$. Production costs are nil.

The problem is that if the first settler to arrive decides for a sustainable use of the land, he must also protect it from other claimants. We assume that there are $n$ claimants, including the first settler. If claimant $i$ expends effort level $x_i$ to appropriate the track of land, he has a probability

$$\frac{x_i}{\sum_{j=1}^{n} x_j}$$

of becoming the owner, in which case he benefits from the sustainable use of the land forever. Assuming that the unit cost of effort is $c$ for all claimants, the *expected* value of the contest for a sustainable use for claimant $i$ is thus

$$V_i = \frac{p(d)y}{r}\frac{x_i}{\sum_{j=1}^{n} x_j} - cx_i. \tag{1}$$

If the first settler opts for mining the land, he does not have to incur any appropriation cost.

a) Assume for now that the first settler decides for a sustainable use of the land. He thus enters into a contest with $n-1$ other claimants. Derive the symmetrical Nash equilibrium level of effort $x_i$ that will be expended by each contestant as a function of $y$, $r$, $c$, $d$ and $n$.
b) Calculate the equilibrium value $V_i^*$ for the first contestant of a sustainable use of the land.
c) Suppose that $n$ is a measure of a country's population size. Compare $V_i^*$ with $p(d)S$ and show that as the population size increases, it becomes less likely that settlers will opt for a sustainable use of land in new settlements.
d) Analyze the effect of distance to market on the type of resource use and appropriation expenditures.

## 3. Regulating fishing season length

A fishery is subject to a free access by $n$ fishers. The government wants to regulate fishing activities by controlling the length of the fishing season. By reducing the fishing period, the *effective* effort of each fisher goes down. Let $x_i$ be the *true* effort of fisher $i$ and $\ell$ be the length of the fishing season, where $\ell$ is expressed in fraction of one year, i.e. $\ell \in (0, 1)$. Effective effort is given by $e_i = \alpha(\ell)x_i$, where $\alpha' > 0$ and $\alpha(1) = 1$. The unit cost of true effort is $c$ and the total steady state harvest function is $h(e)$, where $e$ is the total effective effort, with $h''(e) < 0$.

a) Characterize the Nash equilibrium true effort level for any given season length $\ell$. (Assume that all fishers are identical and find the symmetrical equilibrium.)

b) Characterize the equilibrium when $n \to \infty$. Interpret with the help of a graphic.

Answer the following two questions assuming that $n \to \infty$.

c) Assume $h(e) = ae - be^2$. What is the effect of season length on the total true effort level? On the total effective effort level? On the steady-state resource stock level? On fisher welfare? Discuss.

d) Is it possible to find a fishing season length that would yield an equilibrium at the maximum sustainable yield? Explain.