

## EXERCISES AND PROBLEMS SET 7 (Winter 2007)

### 1. Non-renewable resource exploitation and stock discovery

We wish to compare how a similar increase in the stock of a non-renewable resource can affect its price and extraction paths when it is anticipated versus when it is non-anticipated. Use a four-quadrant graph to analyze this.

- a) In the first case, assume that you are now at time  $t = 0$  and that the stock size is  $S_0$ . The stock is anticipated to increase by amount  $S_1$  at a future specific date, say at date  $t = t_0 > 0$ . (NB Depending on the relative sizes of  $S_0$  and  $S_1$ , there are two cases to consider.)
- b) In the second case, you are now at date  $t = 0$  and the change occurs at that same future time  $t = t_0 > 0$ , but it is a total surprise.
- c) Compare the two cases and interpret.

### 2. A tax on the catch in a steady-state fishery

A fishery is being exploited by a single owner. Total harvesting costs depend on both stock levels and harvesting rate in the following general form:  $C(S(t), h(t))$ , with  $C_1 < 0, C_2 > 0$  and  $C_{22} > 0$ . The unit price of fish is constant and equal to  $p$ . The owner's discount rate is  $r$ . The fish stock varies with time according to the following differential equation:  $\dot{S}(t) = G(S(t)) - h(t)$ . The initial fish stock is  $S_0$ .

- a) Solve for the owner's present value maximizing conditions in steady-state. (Use the Maximum Principle in continuous time.)
- b) What happens when  $r \rightarrow \infty$ ? Interpret.
- c) Assume that the *social* discount rate is equal to  $\rho < \infty$  and that  $r = \infty$ . How can a tax on the catch reestablish a socially optimal stock size?
- d) Characterize the steady-state for an open access exploitation. Compare with your answer in b).
- e) Characterize the tax rate that would reestablish optimality when the fishery is exploited under open access.
- f) Assume now that  $h(t) = eE(t)S(t)$ , where  $E$  is effort and  $e$  is a parameter value related to technology. If the unit cost of effort is  $c$ , then total harvesting cost is now:  $C(S(t), h(t)) = \frac{c}{eS}h = c(S)h$ , with  $c'(S) < 0$ . How do your answers in b) and d) compare?