PROBLEM SET 6 (Fall 2007)

1. Oil depletion, global warming, and Kyoto

Use the *Hotelling rule* to analyze the effects of the following states' interventions aimed at reducing oil consumption. Use a four quadrant graph indicating time, rate of resource extraction, and resource net price. *Interpret* briefly your results. (To simplify, assume zero extraction costs.)

a) The introduction of a worldwide unit tax q on oil. (Hint: For the owner of a resource, a unit tax has the same effect as a constant marginal cost. This must be incorporated in the Hotelling rule.)
b) A subsidy on the use of alternative energy sources. (I leave it up to you to imagine how this would affect the problem.)
c) The introduction of a worldwide unit tax on oil q, with the added twist that the total proceeds from the tax are earmarked for R&D aimed at lowering the cost of the alternative technology.
   i) First, assume simply that the R&D has the effect lowering k over time, i.e. $k = k(t)$ with $\dot{k}(t) < 0$, independently of the tax rate q.
   ii) **(PhD)** Assume now that the higher the tax rate, the faster k decreases over time, i.e. $\frac{\partial}{\partial q}\dot{k}(t) < 0$. Compare the effects of two different tax rates.

2. Pollution discharge in a lake

Without any environmental regulation, a paper mill would discharge a continuous flow K of some pollutant in a lake. With regulation, the flow is reduced to $E(t)$, the value of which depends on the stringency of regulation at instant t. Hence, the reduction in discharge flow is equal to $K - E(t)$. The total cost of such reduction increases in a quadratic way with respect to the magnitude of the reduction, i.e.

$$C(t) = \alpha(K - E(t))^2, \quad 0 \leq E(t) \leq K. \tag{1}$$

Let $S(t)$ be the accumulated stock of pollutant in the lake. The external damage suffered by other users of the lake is a function of this accumulated stock of pollutant and given by

$$D(t) = \gamma S(t)^2. \tag{2}$$

Due to a biological process, some of the pollutant's stock degrades naturally. Hence, the rate of change of the stock of pollutant in the lake is given by:

$$\dot{S}(t) = -\beta S(t) + E(t), \quad X(0) = X_0. \tag{3}$$

This means that discharges $E(t)$ contribute to increase the stock of pollutant and $-\beta S(t)$ denotes the natural degradation process.

The regulator thus faces a trade-off: imposing a lower discharge level $E(t)$ increases the cost of pollution reduction through (1), but it brings a benefit through (2).

a) Solve for the problem of the regulator who must minimize the present value of the sum of pollution reduction and pollution damage costs, i.e. $C(t) + D(t)$. Assume a social discount rate equal to ρ.
b) Interpret the necessary conditions for a maximum.
c) What are the steady-state values for X and E in the optimal solution?

3. Non-renewable resource exploitation and stock size anticipations

Compare how an otherwise similar increase in the stock of a non-renewable resource can affect its price and extraction paths when it is <u>anticipated</u> to when it is <u>not anticipated</u>. (NB The use a four-quadrant graph will help.) In the first case, assume that you are now at time 0 and that the change is anticipated to occur at a future specific date, say at date t_0. In the second case, you are now at date 0 and the change occurs at that same future time t_0, but it is a total surprise. Compare the two cases and interpret.