EXERCISES AND PROBLEMS
SET 6 (Winter 2007)

1. Optimal extraction with extraction costs (From Perman et al., 2003)
A social planner wishes to maximize the discounted stream of instantaneous utility levels expressed as

$$W = \int_0^\infty U(C_t)e^{-\rho t}dt,$$

where C_t is the instantaneous consumption level. The total output is $Q(K_t, R_t)$, where K_t is the stock of capital at time t and R_t is the resource extracted at t, with $Q_K > 0$ and $Q_R > 0$. The capital stock evolves according to

$$\dot{K}_t = Q(K_t, R_t) - C_t - G(R_t, S_t),$$

where $G(R_t, S_t)$ represents the cost of extracting the resource, S_t is the resource stock at t, and with $G_R > 0$ and $G_S < 0$. The resource stock evolves according to

$$\dot{S}_t = F(S_t) - R_t.$$

a) Solve to maximize W and interpret the optimality conditions.
b) Characterize the steady state.
c) How would the problem change if the resource were non-renewable?

2. Continuous-time resource extraction and initial stock size
Go back to the continuous-time non-renewable resource extraction problem seen in class for which $P(R) = ke^{-aR}$.

(1) What are the effects of an increase in the initial stock size S_0 on T, P_t and R_t? Interpret.

3. A renewable resource with existence value (Adapted from Clark, 1976, p. 65)
A community lives next to a renewable resource that has a natural growth rate of $F(x_t)$, where x_t denotes the stock of the resource at time t and $F''(x_t) < 0$. $F'(x_t)$ is initially positive and turns negative after passing the maximum sustainable yield, as is standard for renewable resources (logistic function). The total harvest rate at time t is denoted h_t. The unit harvest cost is $c(x_t)$, with $c'(x_t) < 0$, and the resource sells for a constant unit price p. In addition to its commercial value, the stock of the resource brings some “existence” benefits to the community, which we denote as $V(x_t)$, with $V'(x_t) > 0$. To simplify, we assume that V represents a flow of instantaneous *aggregate* benefits to the community.

a) Solve for the optimal use of the resource as if it were managed as a sole owner by the community, i.e. it maximizes the present value of the sum of commercial and existence benefits as follows:

$$(1) \qquad \max J = \int_0^\infty e^{-\delta t}\{(p - c(x_t))h_t + V(x_t)\}dt$$

Interpret the necessary conditions for a maximum that you obtain. What is the meaning of the shadow-price of x_t?

b) Characterize the steady-state. Show how the presence of existence benefits affect the optimal steady-state stock level?
c) Imagine that the resource is sacred such that any decrease in its stock has a dramatic effect on $V(x)$, i.e. $V'(x)$ is very large. What would be the likely steady-state optimal stock level in that case?
d) Assume now that the resource is non-renewable, i.e. $F(x_t) = 0$. Could you conceive of a steady-state with a positive stock of the resource? Show why or why not.
e) Assume now that the renewable resource is exploited under free access by an *arbitrarily large number* of people that assign no existence value to the resource. Characterize the steady-state equilibrium. What is the effect of increasing the discount rate?

4. Oil depletion, global warming, and Kyoto

Use the *Hotelling rule* to analyze the effects of the following states' interventions aimed at reducing oil consumption. Use a four quadrant graph indicating time, rate of resource extraction, and resource net price. *Interpret* briefly your results. (To simplify, assume zero extraction costs.)

a) The introduction of a worldwide unit tax q on oil. (Hint: For the owner of a resource, a unit tax has the same effect as a constant marginal cost. This must be incorporated in the Hotelling rule.)
b) A subsidy on the use of alternative energy sources. (I leave it up to you to imagine how this would affect the problem.)
c) The introduction of a worldwide unit tax on oil q, with the added twist that the total proceeds from the tax are earmarked for R&D aimed at lowering the cost of the alternative technology.
    i) First, assume simply that the R&D has the effect lowering k over time, i.e. $k = k(t)$ with $\dot{k}(t) < 0$, independently of the tax rate q.
    ii) **(PhD)** Assume now that the higher the tax rate, the faster k decreases over time, i.e. $\frac{\partial}{\partial q}\dot{k}(t) < 0$. Compare the effects of two different tax rates.