PROBLEM SET 5 (Fall 2008)
NON-RENEWABLE RESOURCES

1. Optimal extraction with extraction costs (From Perman et al., 2003)
A social planner wishes to maximize the discounted stream of instantaneous utility levels expressed as

$$W = \int_0^\infty U(C_t)e^{-\rho t}dt,$$

where C_t is the instantaneous consumption level. The total output is $Q(K_t, R_t)$, where K_t is the stock of capital at time t and R_t is the resource harvested at t, with $Q_K > 0$ and $Q_R > 0$. The capital stock evolves according to

$$\dot{K}_t = Q(K_t, R_t) - C_t - G(R_t, S_t),$$

where $G(R_t, S_t)$ represents the cost of extracting the resource, S_t is the resource stock at t, and with $G_R > 0$ and $G_S < 0$. The resource stock evolves according to

$$\dot{S}_t = F(S_t) - R_t,$$

where $F(S_t)$ denotes the natural rate of change of the resource.

a) Solve to maximize W and interpret the optimality conditions.
b) Characterize the steady state.
c) How would the problem change if the resource were non-renewable?

2. Eviction threat and resource extraction
A single firm exploits a non-renewable mineral deposit. The unit selling price p of the resource is constant through time and given for the firm. Per-period total cost of extraction is $C(R_t)$ and displays increasing marginal costs of extraction, i.e. $C'(R_t) > 0$ and $C''(R_t) > 0$. The initial stock of the resource is S_0 and the firm's time *discount factor* is $\beta < 1$.

Due to political instability in the country, the firm faces a threat of eviction at every period. To simplify, suppose that this means that for every period t, the firm assigns a probability π of not being around to exploit the resource at the next period $t+1$ and thereafter. This applies to all period $t = 0, 1, 2, ..., T$.

a) Solve the T-period non-renewable resource extraction problem of the present-value maximizing firm. (NB T is fixed and we assume that the resource constraint is binding.)
b) What happens to the extraction rate when the threat of eviction π increases? Interpret your results.

3. Non-renewable resource exploitation and anticipations over stock size
Compare how an otherwise similar increase in the stock of a non-renewable resource can affect its price and extraction paths when it is anticipated to when it is not anticipated. (NB The use a four-quadrant graph will help.) In the first case, assume that you are now at time 0 and that the change is anticipated to occur at a future specific date, say at date t_0. In the

second case, you are now at date 0 and the change occurs at that same future time t_0, but it is a total surprise. Compare the two cases and interpret.

4. Oil depletion, global warming, and Kyoto

Use the *Hotelling rule* to analyze the effects of the following states' interventions aimed at reducing oil consumption. Use a four quadrant graph indicating time, rate of resource extraction, and resource net price. *Interpret* briefly your results. (To simplify, assume zero extraction costs.)

a) The introduction of a worldwide unit tax q on oil. (Hint: For the owner of a resource, a unit tax has the same effect as a constant marginal cost. This must be incorporated in the Hotelling rule.)

b) A subsidy on the use of alternative energy sources. (I leave it up to you to imagine how this would affect the problem.)

c) The introduction of a worldwide unit tax on oil q, with the added twist that the total proceeds from the tax are earmarked for R&D aimed at lowering the cost of the alternative technology.

   i) First, assume simply that the R&D has the effect lowering k over time, i.e. $k = k(t)$ with $\dot{k}(t) < 0$, independently of the tax rate q.

   ii) **(PhD)** Assume now that the higher the tax rate, the faster k decreases over time, i.e. $\frac{\partial}{\partial q}\dot{k}(t) < 0$. Compare the effects of two different tax rates.