EXERCISES AND PROBLEMS
SET 5 (Winter 2007)

1. Eviction threat and resource extraction
A single firm exploits a non-renewable mineral deposit. The unit selling price p of the resource is constant through time and given for the firm. Per-period total cost of extraction is $C(R_t)$ and displays increasing marginal costs of extraction, i.e. $C'(R_t) > 0$ and $C''(R_t) > 0$. The initial stock of the resource is S_0 and the firm's time *discount factor* is $\beta < 1$.

Due to political instability in the country, the firm faces a threat of eviction at every period. To simplify, suppose that this means that for every period t, the firm assigns a probability π of not being around to exploit the resource at the next period $t+1$ and thereafter. This applies to all period $t = 0, 1, 2, ..., T$.

a) Solve the T-period non-renewable resource extraction problem of the present-value maximizing firm. (NB T is fixed and we assume that the resource constraint is binding.)
b) What happens to the extraction rate when the threat of eviction π increases? Interpret your results.

2. Cake eating (From Perman et al., 2003)
A social planner wishes to maximize the discounted stream of instantaneous utility levels expressed as

$$W = \int_0^\infty U(R_t)e^{-\rho t}dt,$$

where R_t is the instantaneous extraction level. The stock of the resource, S_t, evolves according to

$$\dot{S}_t = -R_t,$$

with the initial stock S_0 given.

(1) Solve to maximize W and interpret the optimality conditions. (Use the Maximum Principle.)
(2) Assume that $U'(R_t) = P(R_t)$, where $P(R_t)$ denotes the net price of the resource which accounts for all its social benefits and costs. What does the optimality condition imply about the rate of change of the net price? Compare with the discrete-time solution seen in class and comment.
(3) What is the effect of increasing the discount rate on the extraction rate?

3. Fishery Dynamics and Market Structure
The natural growth of a fish stock during any year t is given by a logistic function denoted $F(S_t)$, where S_t is the fish stock size in tons at the beginning of year t. With a year t

harvested quantity of H_t tons, the change in stock size between two years is

$$S_{t+1} - S_t = F(S_t) - H_t. \tag{1}$$

The cost of harvest is expressed as $C(H_t, S_t)$, with $C_H(H_t, S_t) > 0$ and $C_S(H_t, S_t) < 0$. The unit selling price p_t of a ton of fish depends on the harvested quantity as per $p_t = p(H_t)$, with $p'(H_t) < 0$. (Note that the demand is constant over time.) The fishery is exploited by a single firm with a discount rate r.

a) Assume that the firm has a monopoly over the fish market and solve for its present-value maximizing problem. Give a "financial interpretation" of your optimality condition. (NB This problem is to be analyzed in discrete time.)
b) How does the monopolist's optimality condition differ from that of a price-taking firm?
c) Give the monopolist's steady-state equilibrium condition?
d) How does the steady-state fish stock of the monopolist compare with that of a price-taking firm? (Hint: Write down the steady-state value $F'(S)$ for the price-taking firm and check whether the monopolist would prefer to increase or decrease that value.)
e) How do monopolists and environmental conservationists get along?