PROBLEM SET 5 (Winter 2007)
NON-RENEWABLE RESOURCES

1. Optimal extraction with extraction costs (From Perman et al., 2003)
A social planner wishes to maximize the discounted stream of instantaneous utility levels expressed as

$$W = \int_0^\infty U(C_t)e^{-\rho t}dt,$$

where C_t is the instantaneous consumption level. The total output is $Q(K_t, R_t)$, where K_t is the stock of capital at time t and R_t is the resource harvested at t, with $Q_K > 0$ and $Q_R > 0$. The capital stock evolves according to

$$\dot{K}_t = Q(K_t, R_t) - C_t - G(R_t, S_t),$$

where $G(R_t, S_t)$ represents the cost of extracting the resource, S_t is the resource stock at t, and with $G_R > 0$ and $G_S < 0$. The resource stock evolves according to

$$\dot{S}_t = F(S_t) - R_t,$$

where $F(S_t)$ denotes the natural rate of change of the resource.

a) Solve to maximize W and interpret the optimality conditions.
b) Characterize the steady state.
c) How would the problem change if the resource were non-renewable?

2. Non-renewable Resource Extraction and the Right to Sell
You are the happy owner of an oil deposit. However, somewhat sadly, you only live for two periods. This means that you have only two periods during which you can extract your oil, periods $t = 0$ and $t = 1$. Due to customary practice, you cannot resell your deposit to anyone else.

a) Let S_0 be the total initial quantity of oil barrels in the deposit. Each barrel fetches a fixed per-period price p_0 and p_1 on the market and the total cost of extraction per period is $c(R_t)$, where R_t denotes the quantity of barrels extracted at period t, with $c'(R_t) > 0$, $c''(R_t) > 0$. Using a discount factor of $\beta = 1/(1 + r)$, write down the present-value profit maximizing problem and derive the corresponding optimal arbitrage condition between R_0 and R_1.
b) Calculate R_0 and R_1 assuming $S_0 = 500$, $p_0 = p_1 = 50$, $r = 10\%$, and $c(R_t) = R^2/20$. Calculate the marginal rent at each period. At what rate is it changing?
c) Do the same exercise as the previous assuming now $S_0 = 1200$. Interpret your result.
d) Suppose now that after you sadly leave us at the end of the period 1, life on Earth fortunately goes on, say for one more period to simplify. The government decides that due to intergenerational equity considerations, you are forced to leave 200 barrels of oil in the ground for the generation living during period $t = 2$, with $p_2 = 50$. Assuming $S_0 = 1200$ still, how would that affect your extraction rates R_0 and R_1?

e) The government considers amending customary law in order to allow you to resell your deposit. We would like to investigate whether the possibility of selling the oil deposit can affect your extraction choices R_0 and R_1, if at all. Analyze this question assuming $S_0 = 1200$ and $p_2 = 50$.

3. Eviction threat and resource extraction

A single firm exploits a non-renewable mineral deposit. The unit selling price p of the resource is constant through time and given for the firm. Per-period total cost of extraction is $C(R_t)$ and displays increasing marginal costs of extraction, i.e. $C'(R_t) > 0$ and $C''(R_t) > 0$. The initial stock of the resource is S_0 and the firm's time *discount factor* is $\beta < 1$.

Due to political instability in the country, the firm faces a threat of eviction at every period. To simplify, suppose that this means that for every period t, the firm assigns a probability π of not being around to exploit the resource at the next period $t+1$ and thereafter. This applies to all period $t = 0, 1, 2, ..., T$.

a) Solve the T-period non-renewable resource extraction problem of the present-value maximizing firm. (NB T is fixed and we assume that the resource constraint is binding.)
b) What happens to the extraction rate when the threat of eviction π increases? Interpret your results.