PROBLEM SET 4 (Fall 2007)

1. A tax on the catch in a steady-state fishery

A fishery is being exploited by a single owner. Total harvesting costs depend on both stock levels and harvesting rate in the following general form: $C(S(t), h(t))$, with $C_1 < 0, C_2 > 0$ and $C_{22} > 0$. The unit price of fish is constant and equal to p. The owner's discount rate is r. The fish stock varies with time according to the following differential equation: $\dot{S}(t) = G(S(t)) - h(t)$. The initial fish stock is S_0.

a) Solve for the owner's present value maximizing conditions in steady-state. *(Use the Maximum Principle in continuous time.)*
b) What happens when $r \to \infty$? Interpret.
c) Assume that the *social* discount rate is equal to $\rho < \infty$ and that $r = \infty$. How can a tax on the catch reestablish a socially optimal stock size?
d) Characterize the steady-state for an open access exploitation. Compare with your answer in b).
e) Characterize the tax rate that would reestablish optimality when the fishery is exploited under open access.
f) Assume now that $h(t) = eE(t)S(t)$, where E is effort and e is a parameter value related to technology. If the unit cost of effort is c, then total harvesting cost is now: $C(S(t), h(t)) = \frac{c}{eS}h = c(S)h$, with $c'(S) < 0$. How do your answers in b) and d) compare?

2. A renewable resource with existence value (Adapted from Clark, 1976, p. 65)

A community lives next to a renewable resource that has a natural growth rate of $F(x_t)$, where x_t denotes the stock of the resource at time t and $F''(x_t) < 0$. $F'(x_t)$ is initially positive and turns negative after passing the maximum sustainable yield, as is standard for renewable resources (logistic function). The total harvest rate at time t is denoted h_t. The unit harvest cost is $c(x_t)$, with $c'(x_t) < 0$, and the resource sells for a constant unit price p. In addition to its commercial value, the stock of the resource brings some "existence" benefits to the community, which we denote as $V(x_t)$, with $V'(x_t) > 0$. To simplify, we assume that V represents a flow of instantaneous *aggregate* benefits to the community.

a) Solve for the optimal use of the resource as if it were managed as a sole owner by the community, i.e. it maximizes the present value of the sum of commercial and existence benefits as follows:

$$\max J = \int_0^{\infty} e^{-\delta t}\{(p - c(x_t))h_t + V(x_t)\}dt \tag{1}$$

Interpret the necessary conditions for a maximum that you obtain. What is the meaning of the shadow-price of x_t?

b) Characterize the steady-state. Show how the presence of existence benefits affect the optimal steady-state stock level?

c) Imagine that the resource is sacred such that any decrease in its stock has a dramatic effect on $V(x)$, i.e. $V'(x)$ is very large. What would be the likely steady-state optimal stock level in that case?
d) Assume now that the resource is non-renewable, i.e. $F(x_t) = 0$. Could you conceive of a steady-state with a positive stock of the resource? Show why or why not.
e) Assume now that the renewable resource is exploited under free access by an *arbitrarily large number* of people that assign no existence value to the resource. Characterize the steady-state equilibrium. What is the effect of increasing the discount rate?