EXERCISES AND PROBLEMS
SET 4 (Winter 2007)

1. Non-renewable Resource Extraction and the Right to Sell

You are the happy owner of an oil deposit. However, somewhat sadly, you only live for two periods. This means that you have only two periods during which you can extract your oil, periods $t = 0$ and $t = 1$. Due to customary practice, you cannot resell your deposit to anyone else.

a) Let S_0 be the total initial quantity of oil barrels in the deposit. Each barrel fetches a fixed per-period price p_0 and p_1 on the market and the total cost of extraction per period is $c(R_t)$, where R_t denotes the quantity of barrels extracted at period t, with $c'(R_t) > 0$, $c''(R_t) > 0$. Using a discount factor of $\beta = 1/(1+r)$, write down the present-value profit maximizing problem and derive the corresponding optimal arbitrage condition between R_0 and R_1.
b) Calculate R_0 and R_1 assuming $S_0 = 500$, $p_0 = p_1 = 50$, $r = 10\%$, and $c(R_t) = R^2/20$. Calculate the marginal rent at each period. At what rate is it changing?
c) Do the same exercise as the previous assuming now $S_0 = 1200$. Interpret your result.
d) Suppose now that after you sadly leave us at the end of the period 1, life on Earth fortunately goes on, say for one more period to simplify. The government decides that due to intergenerational equity considerations, you are forced to leave 200 barrels of oil in the ground for the generation living during period $t = 2$, with $p_2 = 50$. Assuming $S_0 = 1200$ still, how would that affect your extraction rates R_0 and R_1?
e) The government considers amending customary law in order to allow you to resell your deposit. We would like to investigate whether the possibility of selling the oil deposit can affect your extraction choices R_0 and R_1, if at all. Analyze this question assuming $S_0 = 1200$ and $p_2 = 50$.

2. A dynamic, discrete-time analysis of a fishery

Let $G(S_t)$ denote the natural rate of change of a fish stock during period t, where S_t is the fish stock. The harvest function at period t is given by $H_t = eE_tS_t$, where E_t is effort level and e is a productivity parameter. The discount factor is $\beta = 1/(1+r)$, the unit price of fish is p and the unit cost of effort E_t is w.

a) For an infinite number of future periods, characterize the steady-state stock of the resource which corresponds to present value maximization.
b) What is the effect of an increase in productivity parameter e on the steady-state resource stock?
c) What happens when $r \to \infty$?
d) Characterize the open-access steady-state equilibrium. How does it compare to your result in part c?

e) How do your steady-state stock levels in a, c and d compare with the MSY stock level? Interpret.