Economics of Natural Resources
ECO 6143 (Winter 2010)
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PROBLEM SET 4

1. Fishery Dynamics and Market Structure

The natural growth of a fish stock during any year t is given by a logistic function denoted $G(S_t)$, where S_t is the fish stock size in tons at the beginning of year t. With a year t harvested quantity of h_t tons, the change in stock size between two years is

$$S_{t+1} - S_t = G(S_t) - h_t. \tag{1}$$

The cost of harvest is expressed as $C(h_t, S_t)$, with $C_h(h_t, S_t) > 0$ and $C_S(h_t, S_t) < 0$. The unit selling price p_t of a ton of fish depends on the harvested quantity as per $p_t = p(h_t)$, with $p'(h_t) < 0$. (Note that the demand *schedule* is constant over time.) The fishery is exploited by a single firm with a discount rate r.

a) Assume that the firm has a monopoly over the fish market and solve for its present-value maximizing problem. Give a "financial interpretation" of your optimality condition. (NB This problem is to be analyzed in discrete time.)
b) How does the monopolist's optimality condition differ from that of a price-taking firm?
c) Give the monopolist's steady-state equilibrium condition.
d) How does the steady-state fish stock of the monopolist compare with that of a price-taking firm? (Hint: Write down the steady-state value $G'(S)$ for the price-taking firm and check whether the monopolist would prefer to increase or decrease that value.)
e) How do monopolists and environmental conservationists get along?

2. A tax on the catch in a steady-state fishery

A fishery is being exploited by a single owner. Total harvesting costs depend on both stock levels and harvesting rate in the following general form: $C(h(t), S(t))$, with $C_h > 0, C_S < 0$ and $C_{hh} \geq 0$. The unit price of fish is constant and equal to p. The owner's discount rate is r. The fish stock varies with time according to the following differential equation: $\dot{S}(t) = G(S(t)) - h(t)$. The initial fish stock size is S_0.

a) Solve for the owner's present value maximizing conditions in steady-state. *(Use the Maximum Principle in continuous time.)*
b) Assume that the *private* discount rate is very large, i.e. $r \to \infty$, while the *social* discount rate is finite, say $\rho < \infty$. Determine the unit tax on the catch to re-establish a socially optimal stock size.
c) Characterize the steady-state for an open-access exploitation. Determine the unit tax on the catch that would reestablish optimality under open access. Compare with b) and discuss.

d) Assume now that $h(t) = \alpha x(t)S(t)$, where x is effort and α is a given productivity parameter. The constant unit cost of effort is c. Show that the infinite discount rate equilibrium is equivalent to open access. Discuss.

3. A renewable resource with existence value

A community dwells next to a renewable resource. The resource's natural growth rate follows the logistic function $G(S(t))$, with $G''(S(t)) < 0$. The total harvest rate at time t is denoted $h(t)$. The unit harvest cost is $c(S(t))$, with $c'(S(t)) < 0$, and the resource sells for a constant unit price p. In addition to its commercial value, the stock of the resource confers "existence" benefits to the community, which we denote as $V(S(t))$, $V'(S(t)) > 0$. To simplify, we assume that V denotes a flow of instantaneous *aggregate* benefits to the community.

a) Solve for the optimal use of the resource as if it were managed as a sole owner by the community; that is, maximize the present value of the sum of commercial and existence benefits as follows:

$$\max J = \int_0^\infty e^{-\delta t}\{(p - c(S(t)))h_t + V(S(t))\}dt \tag{2}$$

Interpret the necessary conditions for a maximum. What is the meaning of the shadow-price of $S(t)$?

b) Characterize the steady-state. Show how existence benefits affect the optimal steady-state stock level?

c) Imagine that the resource is sacred such that any decrease in its stock size has a dramatic effect on $V(S)$, i.e. $V'(S)$ is very large. What would be the likely steady-state optimal stock level in that case?

d) Assume now that the resource is non-renewable; that is, $G(S(t)) = 0$. Could you conceive of a steady-state with a positive stock of the resource? Show why or why not.

e) Assume now that the renewable resource is exploited under free access by an *arbitrarily large number* of people that assign no existence value to the resource. Characterize the steady-state equilibrium. What is the effect of increasing the discount rate?