Economics of Natural Resources
ECO 6143 (Winter 2010)
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PROBLEM SET 3

1. A basic bio-economic fishery model

a) Suppose that in the absence of fishing activities, the fish stock at time t is given by

$$S(t) = \frac{\bar{S}}{1+\alpha e^{-\rho t}} \text{ where } \alpha \equiv \frac{\bar{S}-S_0}{S_0}. \tag{1}$$

Write the expression for the rate of change $\partial S/\partial t$. Show, through substitutions, that expression (1) implies the following relation between $\dot{S}(t) \equiv \partial S/\partial t$ and $S(t)$:

$$\dot{S}(t) = \rho S(t)\left(1 - \frac{S(t)}{\bar{S}}\right) \tag{2}$$

b) Find the stock sizes at $t = 0$ and $t \to \infty$. How does the growth rate of the resource $\dot{S}(t)/S(t)$ vary with stock size? Where is it maximized? Where is the rate of change maximized? Discuss.

c) Let y denote the harvest rate, assumed to be constant over time. Determine the value $\bar{y}$ which corresponds to the highest sustainable harvest rate in terms of parameter value ρ and $\bar{S}$. How does it compare to $\bar{S}$? What happens when $y = 0.5\rho\bar{S}$? When $y = \rho\bar{S}$?

d) Suppose now that instead of a constant harvest rate, fishers apply a constant effort level, denoted x, say in terms of number of boats. Suppose further that for a given effort level, the harvest depends linearly on the fish stock size, i.e. $y(t) = xS(t)$. Derive the steady-state harvest function, that is, the (sustainable) *yield-effort curve* that can be obtained for any effort level given natural growth equation (2).

e) Verify whether the yield-effort curve is convex or concave. Explain why in light of the fact that harvest function $y(t) = xS(t)$ is linear in x.