PROBLEM SET 3 (Fall 2007)

1. A dynamic, discrete-time analysis of a fishery

Let $G(S_t)$ denote the natural rate of change of a fish stock during period t, where S_t is the fish stock. The harvest function at period t is given by $H_t = eE_tS_t$, where E_t is effort level and e is a productivity parameter. The discount factor is $\beta = 1/(1+r)$, the unit price of fish is p and the unit cost of effort E_t is w.

a) For an infinite number of future periods, characterize the steady-state stock of the resource which corresponds to present value maximization.
b) What is the effect of an increase in productivity parameter e on the steady-state resource stock? What happens when $e \to \infty$?
c) What happens when $r \to \infty$?
d) Characterize the open-access steady-state equilibrium. How does it compare to your result in part c?
e) How do your steady-state stock levels in a, c and d compare with the MSY stock level? Discuss.

2. Grazing land as a renewable resource (discrete time analysis)

The production function for *annual* beef production on a pasture is given by

$$B_t = G_t b(H_t), \quad \text{with } b_H > 0,\ b(0) = 0,\ \lim_{H_t \to \infty} b(H_t) = 1$$

where B_t, G_t, and H_t respectively denote the beef produced (tons), the amount of grazable grass (tons), and the number of beef cattle (heads) allowed to graze. The amount of grass available in year $t+1$ depends solely on the number of beef cattle used the preceding year, i.e.

$$G_{t+1} = g(H_t),$$

with $g'(H_t) < 0$, $g(0) = G_0$, $\lim_{H_t \to \infty} g(H_t) = 0$. Beef sells at a price of p per ton, and the cost of herding, transporting, and processing *each* head of cattle is c.

(1) **Given G_t**, *characterize* the conditions for
  (a) the maximum beef production.
  (b) the number of cattle that will maximize year t's profits.
  (c) The *open access* number of cattle.

  Interpret briefly your results. (We assume that the second-order conditions for a maximum are always satisfied.)

(2) Characterize the conditions for a steady-state equilibrium herd size under open access, year-to-year profit maximization, and maximum sustainable yield. (NB Since this is a discrete-time problem, a steady state implies that $G_{t+1} = G_t$.)

3. Fishery Dynamics and Market Structure

The natural growth of a fish stock during any year t is given by a logistic function denoted $F(S_t)$, where S_t is the fish stock size in tons at the beginning of year t. With a year t harvested quantity of H_t tons, the change in stock size between two years is

$$S_{t+1} - S_t = F(S_t) - H_t. \tag{1}$$

The cost of harvest is expressed as $C(H_t, S_t)$, with $C_H(H_t, S_t) > 0$ and $C_S(H_t, S_t) < 0$. The unit selling price p_t of a ton of fish depends on the harvested quantity as per $p_t = p(H_t)$, with $p'(H_t) < 0$. (Note that the demand is constant over time.) The fishery is exploited by a single firm with a discount rate r.

a) Assume that the firm has a monopoly over the fish market and solve for its present-value maximizing problem. Give a "financial interpretation" of your optimality condition. (NB This problem is to be analyzed in discrete time.)
b) How does the monopolist's optimality condition differ from that of a price-taking firm?
c) Give the monopolist's steady-state equilibrium condition?
d) How does the steady-state fish stock of the monopolist compare with that of a price-taking firm? (Hint: Write down the steady-state value $F'(S)$ for the price-taking firm and check whether the monopolist would prefer to increase or decrease that value.)
e) How do monopolists and environmental conservationists get along?