A common property resource with heterogeneous users

A common-property resource is accessed by two users A and B. The total output is given by quadratic output function

$$f(x) = (2 - x)x,$$

where x denote the sum of individual input effort, i.e. $x = x_A + x_B$. The users differ by the cost of their effort. The respective total costs are given by

$$\begin{aligned} c_A(x_A) &= \frac{1}{2}x_A^2, \\ c_B(x_B) &= x_B^2. \end{aligned}$$

(a) Find the **efficient** allocation of effort x_A^* and x_B^* between the two users. Provide a brief economic interpretation. Calculate the total profit level.

(b) Find the (non-cooperative) Nash equilibrium individual level of effort x_A^{FA} and x_B^{FA} assuming a **free access** regime. Assume that each user's average product of effort is equal to the global average product of effort $f(x)/x$. Calculate the individual profit levels. Compare with the efficient allocation found in (a) and interpret briefly.

(c) Suppose that the users get together in order to assign **non-transferable** quotas on each other's effort level equal to the efficient level, i.e. $q_A^{NT} = x_A^*$ and $q_B^{NT} = x_B^*$. Participation is purely voluntary *ex-ante*. But once it is agreed upon, each user strictly adheres to its quota level, i.e. there is no enforcement problem. Using the free access Nash equilibrium as the benchmark, show that user B will not agree to participate in this scheme. (NB This is essentially equivalent to a non-transferable quota scheme without subsidy.)

(d) Suppose now that **quotas are transferable**. They are initially distributed in the same proportion as the proportion of individual effort that occurs in the non-cooperative free access equilibrium derived in (b), i.e.

$$\begin{aligned} q_A^T &= \frac{x_A^{FA}}{x^{FA}}x^*, \\ q_B^T &= \frac{x_B^{FA}}{x^{FA}}x^*, \end{aligned}$$

where $x^{FA} = x_A^{FA} + x_B^{FA}$ and $x^* = x_A^* + x_B^*$. (This is similar to a *grandfather clause* in which the worst offender actually gets a higher share of quotas.)

i) Show that there are gains from trade such that user A buys $q_B^T - x_B^*$ units of effort quotas from B, thus leading to an efficient allocation of effort. (Hint: You must show that the increase in profit for user A are larger than the drop in profits for user B. Hence there exists a price range for which both will gain from trading $q_B^T - x_B^*$ quotas.)

ii) Show that both users will choose to participate in this scheme once we account for the transfers due to the quotas' price level.

(e) The above results can be **generalized** to common property resource users with heterogeneous characteristics. Discuss the consequences for the possibility of reaching a CPR sharing agreement.

ANSWERS

a) The efficient allocation is found by maximizing the total profit from the resource, i.e.

$$\max_{x_A, x_B} \pi_{TOT} = (2-x)x - \frac{x_A^2}{2} - x_B^2. \tag{1}$$

The FOCs yield

$$x_A^* = 2x_B^* = 2 - 2x^*. \tag{2}$$

Efficiency dictates equality of marginal costs between users and that the marginal costs must be equal to the marginal product. It yields $x_A^* = 0.5$, $x_B^* = 0.25$, $x^* = 0.75$ and $\pi_{TOT}^* = 0.75$.♣

b) The reaction function of user A is found by solving

$$\max_{x_A} \pi_A = x_A(2-x) - \frac{x_A^2}{2}. \tag{3}$$

This yields the following reaction function for A:

$$x_A(x_B) = \frac{2 - x_B}{3}. \tag{4}$$

Similarly for user B,

$$\max_{x_B} \pi_B = x_B(2-x) - x_B^2, \tag{5}$$

implies the reaction function

$$x_B(x_A) = \frac{2 - x_A}{4}. \tag{6}$$

The Nash equilibrium being at the intersection of the two reaction functions, we get $x_A^{FA} = 0.5454$, $x_B^{FA} = 0.3636$, $x^{FA} = 0.909 > x^*$, $\pi_A^{FA} = 0.446$, $\pi_B^{FA} = 0.264$, $\pi_{TOT}^{FA} = 0.71 < 0.75 = \pi_{TOT}^*$.

It is easy to verify that this effort allocation is not efficient for two reasons:

i) Marginal costs are not equal between users, which means that costs are not minimized. More precisely, the marginal cost of user A is below that of user B. Given the output level, it would be better to reduce B's effort and increase A's.

ii) Marginal costs are above the marginal product for both users, which indicates that the resource is being overexploited. This results from the fact that users do not account for the external costs that they impose on each other in terms of lower average productivity of effort.♣

c) We have $q_A^{NT} = x_A^* = 0.5$ and $q_B^{NT} = x_B^* = 0.25$. Which yields $\pi_A^{NT} = 0.5 > \pi_A^{FA} = .446$ and $\pi_B^{NT} = 0.25 < \pi_B^{FA} = 0.264$.

User B prefers the non-cooperative NE outcome since it gives him a higher payoff. This is because user A receives all the benefits from the more conservative use of the resource. User B would have to be compensated to adhere voluntarily to such a scheme. Such compensation is often difficult to implement in practice.♣

d) i) The distribution of quotas is such that

$$q_A^T = \frac{0.5454}{0.909} 0.75 = 0.45 \tag{7}$$
$$q_B^T = \frac{0.3636}{0.909} 0.75 = 0.3 \tag{8}$$

Before quotas are traded, we have

$$\pi_A^T = (2 - 0.75)0.45 - (0.45)^2/2 = 0.4613 \tag{9}$$
$$\pi_B^T = (2 - 0.75)0.3 - (0.3)^2 = 0.285 \tag{10}$$

After quotas are traded, we have

$$\pi_A^* = (2 - 0.75)0.5 - (0.5)^2/2 = 0.5 \tag{11}$$
$$\pi_B^* = (2 - 0.75)0.25 - (0.25)^2 = 0.25 \tag{12}$$

Hence

$$\pi_A^* - \pi_A^T = 0.5 - 0.4613 = 0.0387 \tag{13}$$
$$\pi_B^T - \pi_B^* == 0.285 - 0.25 = 0.035 \tag{14}$$

User A's willingness-to-pay, 0.0387, is above user B's willingness-to-accept, 0.035. Hence, there are gains from trading quotas to reach the optimal allocation of effort.

ii) If the *total* price for quotas is set at the maximum price that A is willing to pay, i.e. $p = 0.0387$, then user A's profits are $\pi_A = 0.4613 > \pi_A^{FA} = .446$. If the *total* price for quotas is set at the minimum price that B is willing to accept, i.e. $p = 0.035$, then user B's profits are $\pi_B = 0.285 > \pi_B^{FA} = 0.264$.

As a result, compared to the free-access situation, both users can gain from the transferable quota scheme with the grandfathering clause. They will thus both participate in such a scheme and efficiency is attained.♣

e) The above two examples with tradeable and non-tradeable quotas illustrate the difficulty involved in finding a sharing rule. Just forcing each participant to reduce use to the efficient level may induce some to refuse participation because they prefer the free-access situation. They will thus ask for direct compensation, which may be difficult to agree on.

Transferable quotas will lead to an efficient outcome while making everyone better off with the grandfathering rule. The problem relates to the initial distribution of quotas. The proportional rule proposed by the grandfathering rule here implies that the "worst abuser" in the free-access regime is actually rewarded with more use rights than the efficient level, with the result that the relatively mild abuser has to buy use rights from the worst abuser. This certainly seems unfair and may make participation difficult to achieve even though all stand to gain.♣