PROBLEM SET 1 (Winter 2022)

1. Non-renewable Resource Extraction and the Right to Sell

Mrs. McBain is the lucky owner of an oil deposit. However, somewhat sadly, she will only live for two more periods and this is known with certainty. (One way to think about this is that each period lasts 30 years and each person lives 90 years. Hence, each person goes through youth (0 to 30 years), middle age (30 to 60 years), and old age (60 to 90 years). Mrs. McBain is now 30 years old.) This means that Mrs. McBain has only two periods left during which she can extract the oil, say periods t = 0 and t = 1. Due to customary practice, you cannot sell the deposit to anyone else.

- a) Let $S_0 = 500$ be the total initial quantity of oil barrels in the deposit. Each barrel fetches a fixed per-period price $p_0 = p_1 = 50$ on the market and the total cost of extraction per period is $C(R_t) = R_t^2/20$, where R_t denotes the quantity of barrels extracted at period t. Mrs. McBain's best alternative investment consists in buying treasury bonds which yield a return r = 10%. Write down the present-value profit maximizing problem and find the corresponding optimal extraction rates R_0^* and R_1^* .
- b) Let us define the marginal rent as $P_t \equiv p_t C'(R_t)$. Calculate the value of the marginal rent at the optimum for each period. At what rate is it changing between the two periods? Interpret.¹
- c) What is the value of increasing the deposit by one extra barrel? Interpret.
- d) Solve the same problem as above assuming now that $S_0 = 1200$. What is the value of increasing the deposit by one extra barrel? Interpret your results with the help of a graph.
- e) Suppose now that after Mrs. McBain sadly leaves us at the end of the period 1, life on Earth fortunately goes on, say for just one more period to simplify. The government decides that due to intergenerational equity considerations, Mrs. McBain is asked to leave 200 barrels of oil in the ground for the generation living during period t = 2, with $p_2 = 50$. Assuming $S_0 = 1200$ still, how would that affect her extraction rates R_0^* and R_1^* ?
- f) The government considers amending customary law such that Mrs. McBain will be allowed to sell her deposit at any time. We would like to anticipate how this can affect her extraction choices R_0^* and R_1^* , if at all. We assume $S_0 = 1200$, $p_t = 50 \ \forall t \in \{0, 1, 2\}$ and

¹By "interpret", I mean that you must explain your results in intuitive economic terms as opposed to just mathematical ones.

r = 10%. (Suggestion: Assume that at the beginning of period 0, Mrs. McBain decides that she will sell her deposit to Frank at the beginning of period 1. You must determine how much Frank will be willing to pay for the remaining stock S_1 and the value he assigns to a marginal barrel.) Is the right to sell good or bad for future generations? Interpret.

2. Land and differential rents with an elastic supply of labor

Differential rents (or Ricardian rents) on land are rents that are generated by variations in land fertility – i.e. its naturally occurring productivity. This may be due to differences in the soil's quality, terrain slope, exposure to rain, etc. It can be analysed in much the same way that was done for the case of locational rents.

Consider three plots of land, A, B and C, with respective output functions $y_i = \alpha_i f(x_i)$, $i \in \{A, B, C\}$, where x_i is the labor input on plot i and α_i is a fertility parameter such that $\alpha_A > \alpha_B > \alpha_C$. Function f is increasing and concave. The unit gate price of the output is the same on all plots and constant throughout. Contrary to our analysis of locational rents, where we assumed a fixed supply of rural labor \bar{x} , we now account for the fact that workers also have the option to work in the urban sector, such that the total rural labor supply is elastic and represented by function x(w), with x'(w) > 0, where w denotes the rural wage rate.

- a) Assume that land owners act as price takers in the labor market and chose labor in order to maximize rents. Write down the equilibrium conditions for the allocation of rural labor between the three land plots. Provide a graphical characterization of the equilibrium and use it to explain the presence of differential rents.
- b) Suppose that the higher fertility of plot A is entirely due to a better exposition to rainfall. In order to solve this problem, an irrigation canal is (freely) built by the government such that the new fertilities of plots B and C, represented by parameters α'_B and α'_C , are now both equal to α_A . The fertility of plot A is unaffected by the canal. Graphically characterize the effect of the canal in the rural sector and discuss its redistributive impact. Would anyone oppose its building?