Problem

Exploiting a copper mine with overlapping generations

Assume a time horizon composed of four periods only, i.e., $t \in \{0, 1, 2, 3\}$. The world ends at the end of period three. Each generation lives for only two periods. There is thus a total of three generations in this problem.

We are now at the beginning of period 0 and Skyler owns a copper mine that she intends to exploit during period 0 and sell to Walter at the beginning of period 1. Walter will exploit the mine during period 1 and sell it to Jesse at the beginning of period 2. Jesse will exploit the mine during periods 2 and 3. Note that Skyler lives for periods $t \in \{0, 1\}$, which "overlaps" with Walter who lives for $t \in \{1, 2\}$, which overlaps in turn with Jesse who lives for $t \in \{2, 3\}$. Skyler and Jesse will therefore never meet. See the timeline in figure 1.

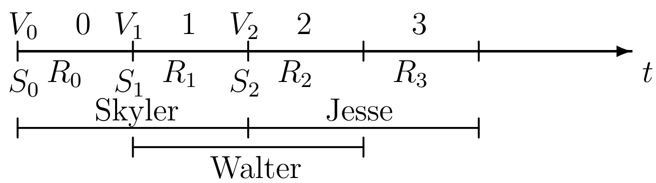

Figure 1: Timeline with overlapping generations

The initial size of the mine is denoted S_0, say in tons of copper. The extracted quantity during period t is denoted R_t. We assume a constant selling price per ton equal to p. The per-period total cost of extraction is given by function $C(R_t)$, with $C' > 0$ and $C'' > 0$. Everyone uses the same discount factor β between periods. We make the following assumptions: each owner seeks to maximize the present value of the mine; the resource constraint is binding; and $R_t^* > 0$, $\forall t$.

a) Represent mathematically the problem that Jesse, Walter and Skyler will be respectively solving.

b) Express the first-order condition for Walter. Interpret by introducing the concept of *user cost*.

c) Show that the problem of Skyler is equivalent to maximizing the present value of the mine by choosing the extraction levels over all four periods.

d) Represent the solution with four graphs side-by-side representing each period, with R_t on the x-axis and the present value of the marginal rent on the y-axis. (Hint: This is quite similar to the locational land rent example seen in class.)

e) With the help of your graphical representation, can you think of a case where $R_3^* = 0$? Interpret.