A sequential two-herder problem

Let $y = f(X)$ denote the total output in a pasture, where X denotes the total number of goats, with $f'(X) > 0$ and $f''(X) < 0$. Two herders have access to the pasture. However, herder 1 gets to choose first the number of goats x_1 to send grazing. Herder 2 chooses his number x_2 after having observed x_1, which is by then fixed. In the game theory literature, we say that herder 1 is the (Stackelberg) *leader* and herder 2 is the *follower*. The unit cost of a goat is equal to c for each herder and its unit selling price is $p = 1$. In order to simplify the calculations, define $\phi(X) \equiv f(X)/X$.

a) Characterize the reaction function of herder 2; that is, his profit maximizing number of goats x_2^f for any given choice x_1. Using the implicit function theorem, determine how x_2^f varies with x_1. (Use the fact that the second-order condition for herder 2 must be respected.)

b) Represent herder 2's reaction function as $x_2(x_1)$ and insert it into the profit maximization problem of herder 1. Solve for herder 1's first-order condition and interpret the result.

c) Assume now that $f(X) = X(a - bX)$ and calculate $x_2'(x_1)$. Using the first-order conditions for each herder, determine x_1^l and x_2^f as functions of parameters a, b and c only. Calculate the Nash equilibrium values x_1^e and x_2^e and compare them with the sequential equilibrium. Discuss your result with the help of the iso-profit curves as illustrated in figure **??**.

d) Compared to the efficient input level X^*, is the free-access equilibrium with sequential moves worse or better than the simultaneous move free-access equilibrium? How about the herders' profit?