ECO6143 Natural-Resource Economics
Mid-term exam March 9 2007
University of Ottawa
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Time allowed: 1h30m

1. Non-renewable resource extraction (50 points)

(a) With the help of a four-quadrant diagram, show how a non-renewable resource should be extracted over time, given a net price curve $P(R)$ with $P'(R) < 0$ and choke price $P(0) = k$, and a discount rate ρ. Explain **intuitively** what is going on. (No formal math necessary.)

(b) Assume that the choke price stays the same but that the demand curve goes down everywhere, i.e. inverse demand rotates down with a pivot at $P(0) = k$. Analyze what happens to the extraction path.

(c) In the discrete time equivalent to part (a), we have seen that price taking firms will also extract at the socially optimal rate, provided that the private and social discount rates coincide. Suppose that this is the case. A new president is now elected in the country with vague intentions to expropriate the firms' oil wells. Firms see this as a probability of *still being there* at future date t to exploit the well equal to $e^{-\lambda t}$. How will this affect the firms' extraction path? Explain.

2. Pollution discharge in a lake (50 points)

Without any environmental regulation, a paper mill would discharge a continuous flow K of some pollutant in a lake. With regulation, the flow is reduced to $E(t)$, the value of which depends on the stringency of regulation at instant t. Hence, the reduction in discharge flow is equal to $K - E(t)$. The total cost of such reduction increases in a quadratic way with respect to the magnitude of the reduction, i.e.

$$C(t) = \alpha(K - E(t))^2, \quad 0 \leq E(t) \leq K. \tag{1}$$

Let $S(t)$ be the accumulated stock of pollutant in the lake. The external damage suffered by other users of the lake is a function of this accumulated stock of pollutant and given by

$$D(t) = \gamma S(t)^2. \tag{2}$$

Due to a biological process, some of the pollutant's stock degrades naturally. Hence, the rate of change of the stock of pollutant in the lake is given by:

$$\dot{S}(t) = -\beta S(t) + E(t), \quad X(0) = X_0. \tag{3}$$

This means that discharges $E(t)$ contribute to increase the stock of pollutant and $-\beta S(t)$ denotes the natural degradation process.

The regulator thus faces a trade-off: imposing a lower discharge level $E(t)$ increases the cost of pollution reduction through (1), but it brings a benefit through (2).

a) Solve for the problem of the regulator who must minimize the present value of the sum of pollution reduction and pollution damage costs, i.e. $C(t) + D(t)$. Assume a social discount rate equal to ρ.
b) Interpret the necessary conditions for a maximum.
c) What are the steady-state values for X and E in the optimal solution?