

**1. Review question (50 points) A renewable resource with existence value**  
 (Adapted from Clark, 1976, p. 65)

A community lives next to a renewable resource that has a natural growth rate of  $F(x_t)$ , where  $x_t$  denotes the stock of the resource at time  $t$  and  $F''(x_t) < 0$ .  $F'(x_t)$  is initially positive and turns negative after passing the maximum sustainable yield, as is standard for renewable resources (logistic function). The total harvest rate at time  $t$  is denoted  $h_t$ . The unit harvest cost is  $c(x_t)$ , with  $c'(x_t) < 0$ , and the resource sells for a constant unit price  $p$ . In addition to its commercial value, the stock of the resource brings some “existence” benefits to the community, which we denote as  $V(x_t)$ , with  $V'(x_t) > 0$ . To simplify, we assume that  $V$  represents a flow of instantaneous *aggregate* benefits to the community.

- (1) Solve for the optimal use of the resource as if it were managed as a sole owner by the community, i.e. it maximizes the present value of the sum of commercial and existence benefits as follows:

$$(1) \quad \max J = \int_0^{\infty} e^{-\delta t} \{ (p - c(x_t))h_t + V(x_t) \} dt$$

Interpret the necessary conditions for a maximum that you obtain. What is the meaning of the shadow-price of  $x_t$ ?

- (2) Characterize the steady-state. Show how the presence of existence benefits affect the optimal steady-state stock level?
- (3) Imagine that the resource is sacred such that any decrease in its stock has a dramatic effect on  $V(x)$ , i.e.  $V'(x)$  is very large. What would be the likely steady-state optimal stock level in that case?
- (4) Assume now that the resource is non-renewable, i.e.  $F(x_t) = 0$ . Could you conceive of a steady-state with a positive stock of the resource? Show why or why not.

**2. Non-renewable resource exploitation (50 points)** Show how a similar increase in the stock of a non-renewable resource can affect its price and extraction paths when it is anticipated and when it is non-anticipated. Use a four-quadrant graph. In the first case, assume that you are now at time 0 and that the change is anticipated to occur at a future specific date, say at date  $t_0$ . In the second case, you are now at date 0 and the change occurs at that same future time  $t_0$ , but it is a total surprise. Compare the two cases and interpret.