

**1. (25 points)** (Adapted from Clark, 1976, p. 65.) A community lives next to a renewable resource that has a natural growth rate of  $F(x_t)$ , where  $x_t$  denotes the stock of the resource at time  $t$  and  $F''(x_t) < 0$ .  $F'(x_t)$  is initially positive and turns negative after passing the maximum sustainable yield, as is standard for renewable resources (logistic function). The total harvest rate at time  $t$  is denoted  $h_t$ . The unit harvest cost is  $c(x_t)$ , with  $c'(x_t) < 0$ , and the resource sells for a constant unit price  $p$ . In addition to its commercial value, the stock of the resource brings some “existence” benefits to the community, which we denote as  $V(x_t)$ , with  $V'(x_t) > 0$ . To simplify, we assume that  $V$  represents a flow of instantaneous *aggregate* benefits to the community.

- (1) Solve for the optimal use of the resource as if it were managed as a sole owner by the community, i.e. it maximizes the present value of the sum of commercial and existence benefits as follows:

$$(1) \quad \max J = \int_0^{\infty} e^{-\delta t} \{ (p - c(x_t))h_t + V(x_t) \} dt$$

Interpret the necessary conditions for a maximum that you obtain. What is the meaning of the shadow-price of  $x_t$ ?

- (2) Characterize the steady-state. Show how the presence of existence benefits affect the optimal steady-state stock level?
- (3) Imagine that the resource is sacred such that any decrease in its stock has a dramatic effect on  $V(x)$ , i.e.  $V'(x)$  is very large. What would be the likely steady-state optimal stock level in that case?
- (4) Assume now that the resource is non-renewable. Could you conceive of a steady-state with a positive stock of the resource? Show why or why not.
- (5) Assume now that the renewable resource is exploited under free access by an *arbitrarily large number* of people that assign no existence value to the resource. Characterize the steady-state equilibrium. What is the effect of increasing the discount rate?

**2. (25 points)**

- (1) It is often argued that private ownership solves the problem of resource over-exploitation. Explain why this may not always be true. Be as complete as possible.
- (2) Give some reasons why the problem of the tragedy of the commons may not be a good representation of the way people exploit resources in common. Be as complete as possible.
- (3) Why is it that state intervention can lead to a worse outcome than the pre-existing state of affairs in some common property resources?