0.0.1 Understanding property rights through the lens of game theory

Despite the many insights that one can already draw from Gordon's (1954) analysis, we will now see that one can learn a great deal more on the role played by property right arrangements through the application of some basic tools of game theory. To this end, we begin with the simple example of the two-herder problem that Hardin (1968) used to illustrate what is termed the *tragedy of the commons*. The game-theoretical representation proposed below is however based on Cheung (1970) and Dasgupta and Heal (1979).

Let us begin by describing a production technology on a natural asset, that of a certain area of pasture land. Assume that this area can only be used for the purpose of raising goats for the production of meat. This area being of fixed size, it will not appear explicitly as an input variable in our representation of the production technology. We assume that the only other input for meat production takes the form of the number of goats sent grazing, a chosen quantity. Hence, if X denotes the total number of grazing goats and y the total output produced in, say, kilograms of meat, we have $y = f(X)$, with function f assumed to be increasing and concave. We simplify by assuming that production decisions are made only once in a one-period, static problem. We therefore abstract from the dynamic stock-flow effects inherent in pasture land use issues, such as grass growth or soil erosion.

We now turn to the *circumstances* that surround the actual choice of input quantity X. It goes without saying that this choice will be affected by prices. Here, to make things simple, we assume that each grazing goat requires a constant unit cost c and that each kilogram of meat fetches a constant unit price p. But the choice of X does not depend only on prices; it also depends on the *institutional* circumstances, described as follows.

Suppose that only two herders are allowed to access the pasture with goats. We consider a simple setting in which the herders decide only once on their input quantities. Each herder can freely access the pasture with goats in the sense that no restriction is being imposed. We refer to this type of institutional arrangement as a free-access property regime, more precisely defined as follows:

Definition 1 *A* ***free-access property regime*** *refers to a situation where (i) a well-defined group of users have exclusive access to a resource and (ii)* *no use restriction is being imposed on the individual user.*

In the present case, the group of users is made up of just two herders. The herders are assumed identical in the sense that each unit of input is equally productive in producing output units. Consequently, if we let $\phi(X) \equiv f(X)/X$ denote the average product of a goat, then the total output received by herder i is given by $x_i\phi(X)$, where x_i denotes the number of goat input chosen by herder i, $i \in \{1, 2\}$. It is important to keep in mind that with two herders present, the total input quantity is given by $X = x_1 + x_2$.

In order to apprehend the implications of a free-access regime, the use of the *Nash equilibrium* concept is an extremely insightful analytical tool. Recall first that a Nash equilibrium posits that decisions are made in a *non-cooperative* fashion. This is appropriate because the fact that no restriction is being imposed on any user suggests that use decisions are made non-cooperatively. Indeed, if the herders were to cooperate, they would have to set rules of use and adhere to them somehow. Cooperation is thus tantamount to imposing use restrictions on each other and violates our definition of a free-access property regime. A Nash equilibrium further assumes that decisions be made simultaneously, without previous communication, and that the game is played only once. These assumptions are made for simplicity now and could be relaxed to make the analysis more realistic, though at the cost of higher complexity. As will be seen below, our simplified representation already yields much useful insights.

Recall that a Nash equilibrium requires that each herder's choice be on his or her *reaction function*, i.e., if (x_1^e, x_2^e) constitutes a Nash equilibrium choice of input quantities, then x_1^e maximizes herder 1's gain for given x_2^e, and analogously for herder 2's choice. In order to find herder 1's reaction function, we must solve for the choice x_1 that maximizes his gain for any *given* choice x_2, as per the following problem:

$$\max_{x_1} \pi_1 = x_1 p\phi(X) - x_1 c \text{ where } X = x_1 + x_2,\ x_2 fixed. \tag{1}$$

The first-order condition for an interior solution is

$$\frac{\partial \pi_1}{\partial x_1} = p\phi(X) + x_1 p\phi'(X) - c = 0. \tag{2}$$

This condition tells us that at the margin, when herder 1 adds a goat to the pasture, the total effect on profits can be decomposed into three parts:

term $p\phi(X)$ indicates that profits increase by the average product of a goat; term $x_1 p\phi'(X)$ denotes a cost in terms of a drop in average productivity that affects all input quantities x_1; and there is a direct unit cost c. Given x_2, herder 1 will increase x_1 up to the point where the marginal benefit is exactly offset by the sum of the two types of costs. Note that with decreasing returns, the average product $\phi(X)$ decreases with x_1. Hence, if one assumes that as X becomes arbitrarily large, the average product tends to zero - i.e., $\lim_{X\to\infty}\phi(X) = 0$ - then it will eventually fall below c. There must therefore be at least one value of x_1 for which condition (2) is respected. In exercise 1, the reader is asked to verify under what conditions the second-order condition is respected.

Exercise 1 *Verify whether the second-order condition for a unique maximum to the herder problem in (1) are respected. Are they respected by the quadratic production function $f(X) = aX - bX^2$?*

The three-part decomposition of first-order condition (2) turns out to be extremely useful for our understanding of the fundamental role that property right arrangements play in explaining the presence of a *negative externality*. Indeed, in the case of the free-access regime here, even though the negative productivity effect $\phi'(X)$ in the middle term affects both users' inputs, herder 1 only accounts for this loss to the extent that it affects her own input quantity x_1, while ignoring the losses it imposes on herder 2's input effort x_2. Let us see how this leads to resource overuse.

Condition (2) implicitly defines a relation between x_1 and x_2 in the sense that for any given x_2, there is a value of x_1 that satisfies the equality. To see how, let us define the following function: $\psi(x_1, x_2) \equiv \phi(X) + x_1\phi'(X) - c$. Its total differential is

$$d\psi = \frac{\partial \psi}{\partial x_1} dx_1 + \frac{\partial \psi}{\partial x_2} dx_2. \tag{3}$$

Take care to review the reasoning behind this expression. The first term on the right-hand side says that if x_1 increases by marginal value dx_1 then, in order to measure its impact on ψ, one must multiply this increase by the slope of ψ with respect to x_1. And similarly for dx_2. The sum of these two effects gives the total change in ψ. Now first-order condition (2) requires that ψ remain constant and equal to zero, i.e., following a change in x_2, herder 1 always adjusts x_1 in such a way that $d\psi = 0$ in (3). Accounting for this and rearranging slightly yields:

$$\frac{dx_1}{dx_2} = -\frac{\psi_{x2}}{\psi_{x1}}. \tag{4}$$

Some readers may have noticed that the above expression constitutes an application of the *implicit function theorem*. This method provides a powerful way to look at how an endogenous variable varies with respect to an exogenous variable, given some derived equilibrium condition. For instance, expression (4) tells us that x_1 decreases with x_2. Indeed, the denominator ψ_{x1} must be negative because it corresponds to the second-order condition to problem (1). The sign of the numerator, $\psi_{x2} = \phi'(X) + x_1\phi''(X)$, is not so well defined since we did not make any specific assumption about the sign of term $\phi''(X)$. Now given that $\phi'(X) < 0$, we shall assume that $\psi_{x2} < 0$. (A sufficient condition is that $\phi''(X) \leq 0$. If not, then we must require that the magnitude of $\phi''(X)$ not be too large in absolute terms.) In exercise 2, the reader is asked to verify this result with the help of a graphical analysis.

Exercise 2 *Plot the profit function of herder 1 as a function of x_1 for a given value $x_2^0 > 0$. Identify the value $x_1(x_2^0)$ that maximizes π_1. Do the same with another value x_2^1 such that $x_2^1 > x_2^0$. Use that graphic to verify that $x_1(x_2)$ decreases with x_2.*

As intuition would dictate, therefore, an increase in x_2 induces herder 1 to decrease the quantity of goats she uses. In the language of game theorists, inputs are said to be *strategic substitutes*, in the sense that an increase in one player's choice variable induces the other to reduce her's.[1] The reaction function of herder 1 as defined by (2) is denoted $x_1(x_2)$. That of herder 2 is derived analogously and is denoted $x_2(x_1)$. Both curves are illustrated in figure 1, along with the profile of three iso-profit curves for herder 1, denoted π_1^A, π_1^B and π_1^C. The precise manner in which the reaction functions are drawn is left as exercise 3.

Exercise 3 *Use expressions (2) and (4) to verify that: (i) the reaction curves can cross only once; (ii) $x_2(x_1)$ crosses $x_1(x_2)$ from above; (iii) $x_i(0) = X^*$, where X^* is the efficient input level in (**??**); and (iv) $x_i(x_j) > 0 \forall x_j \in [0, X^{OA})$ and $x_i(x_j) = 0 \forall x_j \geq X^{OA})$, where X^{OA} is the rent dissipating input level introduced in section **??**.*

[1] See, for instance, Tirole (1988).

Exercise 4 *Use the envelop theorem to show that herder 1's profits decrease with x_2 along the reaction function. Use that fact to argue that the iso-profit curves must be shaped as illustrated in figure 1.*

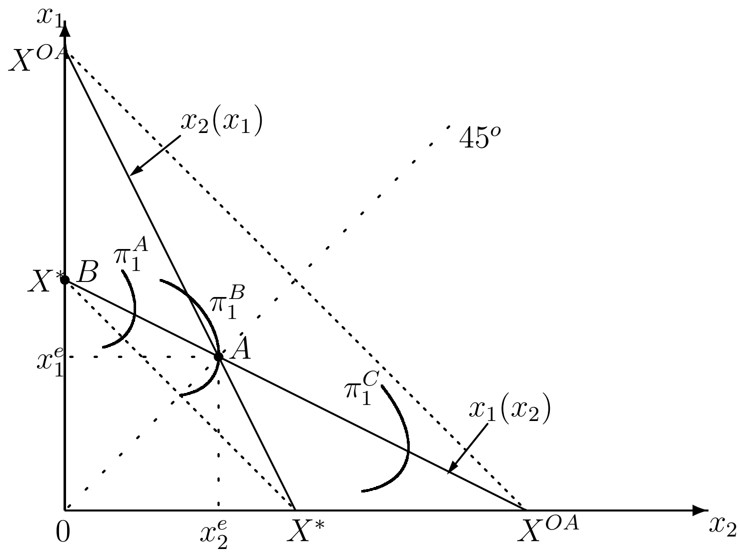

Figure 1: The herders' reaction functions and the Nash equilibrium

The Nash equilibrium is therefore given by coordinate point (x_1^e, x_2^e) where the reaction curves intersect at point A. As is generally the case with this type of analysis, we are especially interested in knowing how this equilibrium compares with the efficient allocation; here, this means comparing $x_1^e + x_2^e$ with X^*. One can easily show that $x_1^e + x_2^e > X^*$. Indeed, dotted line X^*X^* represents an iso-input line along which $x_1+x_2 = X^*$. Since the Nash equilibrium is located above that line, we conclude that the free-access equilibrium with two users leads to resource overuse. But how does the equilibrium compare with the rent dissipating one? To answer this, note that dotted line $X^{OA}X^{OA}$ represents points with total input equal to the open access level, i.e., $x_1 + x_2 = X^{OA}$, and lies strictly above point A. Hence, the free access total input level is lower than the rent dissipating one. We therefore have $X^* < x_1^e + x_2^e < X^{OA}$. Consequently, although total rents are not maximized under a free-access regime, they are not entirely dissipated. Indeed, one can readily see from figure **??** that under the free access equilibrium with two herders, the average product exceeds the average cost. In what way does the free access regime, then, differ from the open access situation discussed in section **??**? In order to answer this, let us go back to our previous discussion about negative externalities.

Exercise 5 *Verify through graphical analysis that the Nash equilibrium in figure 1 is stable. How important is it for stability that* $x_2(x_1)$ *crosses* $x_1(x_2)$ *from above?*

In the open-access setting of section **??**, we assumed that outside fishers, when considering to enter into the fishery or not, simply compared the average product value of an additional unit of effort with its unit cost. This argument implies that each additional fisher is not at all concerned about the effect this decision has on the average product. This contrasts with the two-herder, free-access setting where each herder i is worried about how an additional input negatively affects the average product of his own inputs, as per term $x_1 p\phi'(X)$ in expression (2). Given this additional cost, it is not surprising that this leads to a less severe overuse of the resource. Does that mean that Gordon's (1954) analysis was flawed? No really. We will argue in section **??** that one can reconcile the two models by showing that the open-access regime constitutes a special case of the free-access one in which the number of users is arbitrarily large.

But if the two herders do account for the negative productivity effects, why do they still overexploit the pasture? The answer is again provided by term $x_1 p\phi'(X)$ in expression (2). Indeed, herder 1 is only concerned about the productivity effects that affect his own inputs x_1 while ignoring those imposed on herder 2. This external effect is valued at $x_2 p\phi'(x_2)$. With herder 1's costs being lower than the true "social" cost of adding a goat, it is not surprising to observe that the free-access regime leads to over-exploitation. Gordon's (1954) prescription was therefore to give the full control of the pasture to one entity that would receive the full benefits from input use and bear the full costs, in which case term $x_1 p\phi'(X)$ in expression (2) changes to $Xp\phi'(X)$.

The foregoing analysis highlights the importance of the interaction between institutions and technology. In the above setting, property rights play a role to the extent that the technology of production exhibits decreasing returns to input efforts. Indeed, with constant average products, the action of one herder does not affect the productivity of another and as a consequence, the ability to exclude users brings no benefit. (The reader is encouraged to verify this in exercise **??**.)

The analysis also provides insights on the link between property regimes and negative externalities. In the foregoing free-access, two-herder model, the drop in average productivity causes a negative externality because the productivity of herder 2's goats is not accounted for by herder 1. If, however, herders 1 and 2 get together to form a unitary decision-making body about the total number of goats to send grazing, then this entity will account for the drop in productivity of herder 2's goats caused by herder 1. The creation of such a unit is tantamount to the creation of exclusive property rights over the whole pasture in the sense that one unit has full control over access to the resource and receives all the benefits. This simple change has caused negative externalities to disappear even though herder 1's goats are still affecting the productivity of herder 2's. Hence, the fact that average products decrease as the number of inputs increase is a purely technological effect that is present regardless of the property regime.

The above model yields an equilibrium in which both users have herds of equal sizes. Johnson and Libecap (1980) present cases where large asymmetries between herders' stock sizes were observed among the U.S. southwestern Indian reservations. With the introduction of sequential moves, the following problem yields an equilibrium that is consistent with the presence of asymmetric input efforts by users.[2]

[2] It should be noted that Johnson and Libecap (1980) emphasize the role of decreasing average costs as being responsible for the asymmetries. The sequential move model suggests that this is not necessary while still being consistent with the idea that a first mover will overexploit in order to induce lower input use by the follower.