(N.B. This question is virtually identical to assigned exercise #5.10.) 7.1

MID-TERM 2006

QUESTION 7: ① Solution based on cost function

$$\pi = py - c(\vec{w}, y)$$

$$\Rightarrow \frac{\partial \pi}{\partial y} = p - \frac{\partial}{\partial y} c(\vec{w}, y^*) = 0$$

$$\Rightarrow \frac{\partial}{\partial y} c(\vec{w}, y^*) = p.$$

Let $\Psi(\vec{w}, y^*) \equiv \frac{\partial}{\partial y} c(\vec{w}, y)$

$$\Rightarrow d\Psi = \frac{\partial}{\partial w_1} \frac{\partial}{\partial y} c(\vec{w}, y) dw_1 + \frac{\partial}{\partial w_2} \frac{\partial}{\partial y} c(\vec{w}, y) dw_2$$

$$+ \frac{\partial^2}{\partial y^2} c(\vec{w}, y) dy = 0$$

$$\Rightarrow \frac{\partial}{\partial y}\left[\frac{\partial}{\partial w_1} c(\vec{w}, y) dw_1 + \frac{\partial}{\partial w_2} c(\vec{w}, y) dw_2\right] = -\frac{\partial^2}{\partial y^2} c(\cdot) dy$$

Using Shephard's lemma and rearranging, we get:

$$dy = -\frac{\frac{\partial}{\partial y}\left[x_1(\vec{w}, y) dw_1 + x_2(\vec{w}, y) dw_2\right]}{\frac{\partial^2}{\partial y^2} c(\vec{w}, y)}$$

By the S.O.C., we have $\frac{\partial^2}{\partial y^2} c(\vec{w}, y) > 0$.

Hence $dy > 0$ iff

$$\frac{\partial}{\partial y} x_1(\vec{w}, y)\, dw_1 + \frac{\partial}{\partial y} x_2(\vec{w}, y)\, dw_2 < 0.$$

This implies that one of the two input types must be an *inferior factor*. It means that in order to produce more output, the firm actually *chooses* to use less of that input. As we have seen, this leads to the paradoxical possibility that the output increases after an increase in all input prices.

② Solution based on supply function

Let $y(w_1, w_2, p) \equiv$ supply function

$$\Rightarrow dy = \frac{\partial y(\vec{w}, p)}{\partial w_1} dw_1 + \frac{\partial y(\vec{w}, p)}{\partial w_2} dw_2 \quad \text{(BY DEF.)}$$

$$= -\frac{\partial x_1(\vec{w}, p)}{\partial p} dw_1 - \frac{\partial x_2(\vec{w}, p)}{\partial p} dw_2$$

(BY SYMMETRY OF SUBST. MATRIX)

$$= -\frac{\partial x_1(\vec{w}, y)}{\partial y} \frac{\partial y(\vec{w}, p)}{\partial p} dw_1 - \frac{\partial x_2(\vec{w}, y)}{\partial y} \frac{\partial y(\vec{w}, p)}{\partial p} dw_2$$

(SINCE $x_i(\vec{w}, p) = x_i(\vec{w}, y(\vec{w}, p))$)

$$\Rightarrow dy = \left[-\frac{\partial x_1(\vec{w}, y)}{\partial y} dw_1 - \frac{\partial x_2(\vec{w}, y)}{\partial y} dw_2 \right] \frac{\partial y(\vec{w}, p)}{\partial p}$$

$\Rightarrow dy > 0$ iff one factor is such that

$$\frac{\partial x_i(\vec{w}, y)}{\partial y} < 0$$

$\Rightarrow$ It is an inferior factor.

<u>Q:</u> Can both factors be inferior?