Mid-Term Exam
Micro IV - ECO6122
FALL 2007
PART A

1a) A cost function gives the minimum cost of producing a given output level (y) at fixed factor prices $(\vec{w})$. It can be expressed as:

$$c(\vec{w}, y) = \min_{\vec{x}} \vec{w}\vec{x} \quad s.t. \; f(\vec{x}) = y$$

where $f(\vec{x})$ is the production fctn.

1b) See demo. in Varian p. 67.

REMARK: This was a difficult question as I don't expect students to know all the proofs. However, I do expect that they go through it once. So I am looking at the procedure. Two important features are:

① You are trying to show that $c(\vec{w}, y) = y\,c(\vec{w}, 1)$

② The proof must make use of the CRS properties of the technology.

1c) $AC = \dfrac{c(\vec{w}, y)}{y}$ and $MC = \dfrac{\partial}{\partial y} c(\vec{w}, y)$

Since $c(\vec{w}, y) = y\,c(\vec{w}, 1)$, we have

$AC = MC, \; \forall y.$

1d) A fixed cost must be paid independently of the output level.

Let $TC = c_v(\vec{w}, y) + F$

where F is the fixed cost.

With CRS, we must have $TC = y\,c(\vec{w}, 1)$

$\Rightarrow c_v(\vec{w}, y) + F = y\,c(\vec{w}, 1)$, $\forall y$.

$\Rightarrow \frac{\partial}{\partial y} c_v(\vec{w}, y) = c(\vec{w}, 1)$ (a constant)

$\Rightarrow c_v(\vec{w}, y) = y\,c(\vec{w}, 1)$

Since $y\,c(\vec{w}, 1) = TC$, $F = 0$.

There is no fixed cost with a CRS technology.

REMARK: Some have rather tried to show that CRS becomes DRS when some factors become fixed, which is OK also.

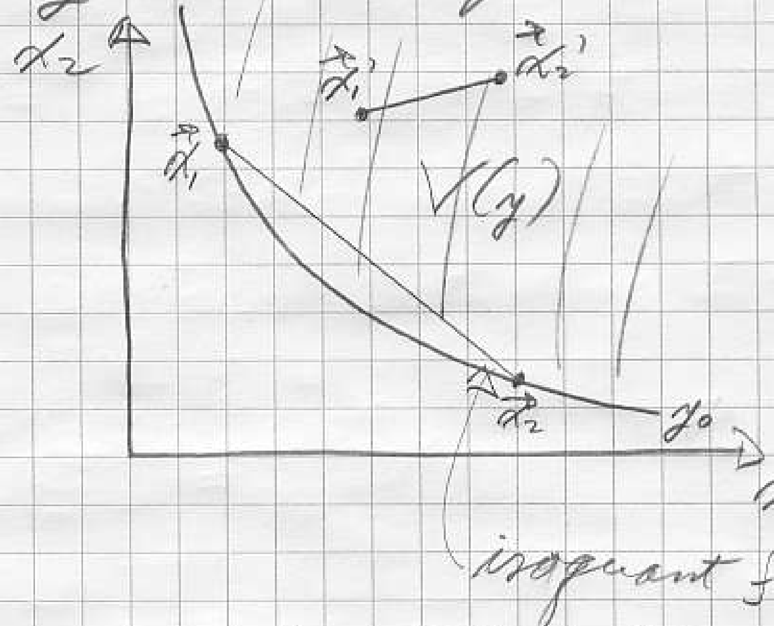
2a) Take $\vec{x}_1 \in V(y)$ and $\vec{x}_2 \in V(y)$, where $V(y)$ is the input requirement set.

$V(y)$ is convex if

$\vec{x}_t = t\vec{x}_1 + (1-t)\vec{x}_2 \in V(y)$, $\forall t \in (0,1)$.

Any convex combination of two points in $V(y)$ remains in $V(y)$.

Graphically, this implies



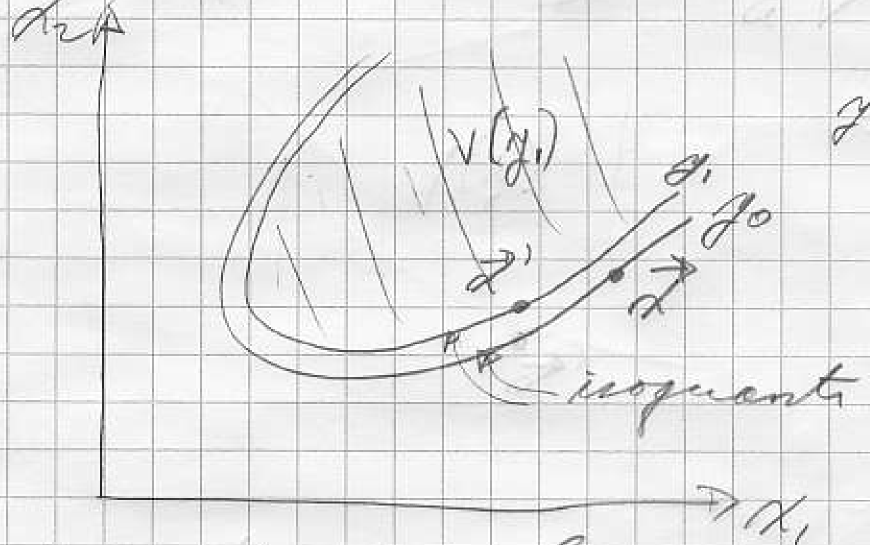
NB You must identify all axes and curves.

2b) Convexity of $V(y)$ means that one can produce at least as much with averages than extremes.

or: Another way to say this is that it becomes harder and harder to substitute one input for another the input ratio x_1/x_2 becomes very large or very small.

or: Others have mentioned the replication argument, which is OK also.

2 c) <u>YES</u>. Example:



This $V(y)$ is not monotonic at input combination $\vec{x}$ since $\vec{x} > \vec{x}'$ and $y_0 < y_1$. However, $V(y)$ is convex.

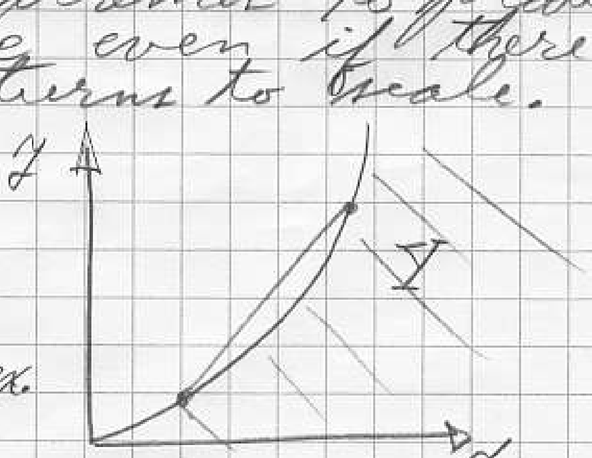
2 d) <u>YES.</u>

Convexity of PPS Y means that there is <u>no</u> increasing returns to scale.

Convexity of $V(y)$ means that averages are better than extremes to produce. This can be true even if there are increasing returns to scale.

ex: $f(x) = x^2$

$\Rightarrow$ Y not convex.



$V(y) = \{x \mid x > \sqrt{y}\}$ is convex.