October 28th 2008

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa
Mid-term exam
Professor: Louis Hotte

NB This exam has 2 pages. Some formulae are provided in the APPENDIX (that you may or may not need).

Part A. Review questions (60 points)

A1. (20 points) True or False: The expenditure function is homogenous of degree 1 in $\overrightarrow{p}$. Demonstrate.

A2. (20 points) A cost function has the form $c(w, y) = Aw_1^{\alpha} w_2^{\beta} y$.

a) Find the conditional input demands.

b) Show that the input shares s_i as the proportion of total expenditure are independent of the scale of production.

A3. (20 points) Demonstrate the following property of demand (Engel aggregation):

$$\sum_{i=1}^{n} s_i \eta_i = 1, \qquad (1)$$

where $s_i = p_i x_i / y$ and η_i is income elasticity of demand for good i.

Part B. questions (40 points)

B1. (20 points) True or False: The conditional input demand function is homogeneous of degree 0 in prices and output $(\overrightarrow{w}, y)$. Justify your answer. (Suggestion: Use a graphic with two types of goods, i.e. $n = 2$.)

B2. (20 points)
A consumer in a three-good economy has the following Marshallian demand functions for goods 1 and 2:

$$x_1(\overrightarrow{p}, y) = 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{y}{p_3}, \qquad (2)$$

$$x_2(\overrightarrow{p}, y) = \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3} + \delta\frac{y}{p_3}. \qquad (3)$$

a) Indicate how you would calculate the demand for good 3 (do not solve for it).

b) Verify whether the demand functions $x_1(\overrightarrow{p}, y)$ and $x_2(\overrightarrow{p}, y)$ are appropriately homogeneous.

c) Show that it must be the case that $\alpha = 100$ and $\beta = \gamma = -5$.

d) For fixed x_3, draw the indifference curve in the x_1, x_2 plane.

e) Show that it must be the case that $\delta = 0$.

f) Given the above, try to determine how the utility function $u(x_1, x_2, x_3)$ is like.

APPENDIX

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v / \partial p_i}{\partial v / \partial y}$$

$$p_i(\overrightarrow{x}) = \frac{\partial u / \partial x_i}{\sum_{j=1}^{n} x_j (\partial u / \partial x_j)}$$