15.1 MATCHING PENNIES:

Let $p_H^C \equiv$ proba. that COLUMN plays HEADS

The problem of Row is:

$$\max_{p_H^R} V^R = p_H^R\left[p_H^C \cdot 1 + (1-p_H^C)\cdot(-1)\right] + (1-p_H^R)\left[p_H^C\cdot(-1) + (1-p_H^C)(1)\right]$$

where $p_H^R \equiv$ proba. that ROW assigns to strategy HEADS.

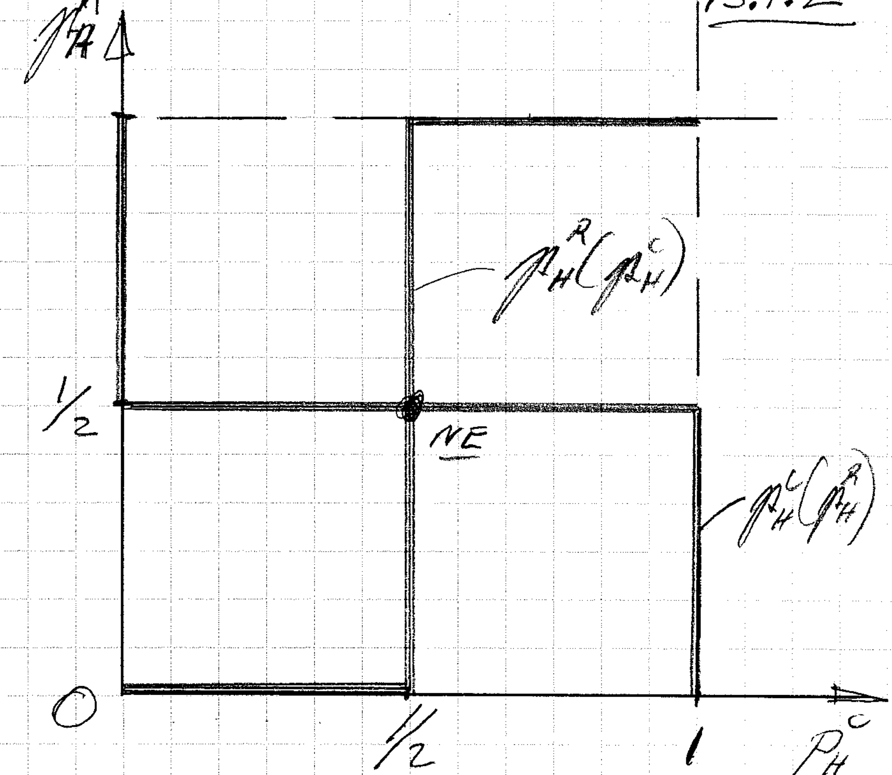
We have

$$\frac{\partial V^R}{\partial p_H^R} = p_H^C - 1 + p_H^C + p_H^C - 1 + p_H^C = 4p_H^C - 2$$

$$\frac{\partial^2 V^R}{\partial {p_H^R}^2} = 0 \Rightarrow \text{Corner solutions:}$$

$$\frac{\partial V^R}{\partial p_H^R} \begin{cases} < 0 \text{ iff } p_H^C < 1/2 \Rightarrow p_H^{R*} = 0 \\ = 0 \text{ iff } p_H^C = 1/2 \Rightarrow p_H^{R*} \in [0,1] \text{ (indifference)} \\ > 0 \text{ iff } p_H^C > 1/2 \Rightarrow p_H^{R*} = 1 \end{cases}$$

BEST-RESPONSE CURVES:



Given p_H^R, the problem of COLUMN is:

$$\max_{p_H^C} V^C = p_H^C\left[p_H^R(-1) + (1-p_H^R)(1)\right] + (1-p_H^C)\left[p_H^R(1) + (1-p_H^R)(-1)\right]$$

$$\Rightarrow \frac{\partial V^C}{\partial p_H^C} = -p_H^R + 1 - p_H^R - p_H^R + 1 - p_H^R = 2 - 4p_H^R$$

$$\Rightarrow \frac{\partial V^C}{\partial p_H^C} \begin{cases} < 0 & \text{iff } p_H^R > 1/2 \Rightarrow p_H^{C*} = 0 \\ = 0 & \text{iff } p_H^R = 1/2 \Rightarrow p_H^{C*} \in [0, 1] \\ > 0 & \text{iff } p_H^R < 1/2 \Rightarrow p_H^{C*} = 1 \end{cases}$$

The best-response curves indicate that there is only one Nash equilibrium defined at $p_H^C = p_H^R = 1/2$.

15.6 THE CHICKEN GAME

a) Pure strategy Nash equil.

		II: L	II: R
I	T	-3, -3	$\underline{2}, \underline{0}$ *
	B	$\underline{0}, \underline{2}$ *	1, 1

There are two pure strategy N.E.:

(B, L) & (T, R)

b) Mixed strategy NE:

Define $p_L \equiv$ proba. that player II plays L.

i) Player I strictly prefers T over B iff:

$$p_L(-3) + (1-p_L)(2) > p_L(0) + (1-p_L)(1)$$

$$\Rightarrow \quad -5p_L + 2 > 1 - p_L$$

$$\Rightarrow \quad p_L < 1/4$$

He is indifferent iff $p_L = 1/4$

He strictly prefers B over T iff $p_L > 1/4$

ii) Player II has a strict preference for L iff

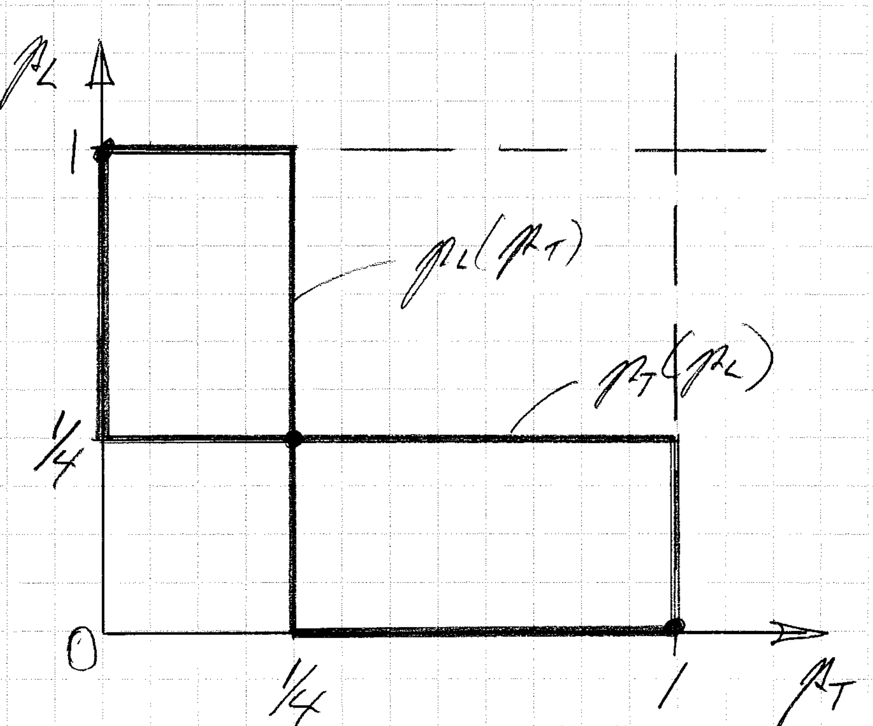
$$p_T(-3) + (1-p_T)(2) > p_T(0) + (1-p_T)(1)$$

$$\Rightarrow \quad p_T < 1/4$$

where p_T ≡ proba. that player I chooses T.

Player II is indifferent between L and R iff $p_T = 1/4$.

And he prefers strictly R iff $p_T > 1/4$.



There is a NE in mixed strategy at $p_L = p_T = 1/4$.

Note that we also have the two pure strategy NE: $p_T = 0$ with $p_L = 1$

$p_T = 1$ with $p_L = 0$.

c) With both pure strategy NE, both players survive with certainty.

With the mixed-strategy NE, there is a $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ proba. that both choose to stay and get killed.

Oboiously, there is a problem of multiple equilibria with this game. We cannot predict which of the three N.E. the players will choose to concentrate on.

Repeated symmetric duopoly

$\Pi_j \equiv$ Cartel payoff per firm (collusion)

$\Pi_c \equiv$ Cournot competition payoff per firm.

$\Pi_d \equiv$ Maximum payoff for one firm given that the other firm has chosen the cartel output level.

$\Pi_d > \Pi_j > \Pi_c$

Trigger strategy for infinitely repeated game:

① Choose cartel output level if both firms have chosen cartel output level in the past

② Choose Cournot output level otherwise.

We want to verify if the cartel output can be maintained.

Assuming that both firms have chosen the cartel output level in the past, the present value of sticking to the trigger strategy for firm 1, given that firm 2 does, is:

$$V_1 = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \Pi_j = \frac{1+r}{r} \Pi_j$$

If firm 1 deviates from the trigger strategy, its present value is:

$$V_1' = \Pi_d + \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t \Pi_c = \Pi_d + \frac{1}{r} \Pi_c$$

In order for the cartel output be sustained, we must have $V_1 \geq V_1'$

$$\Rightarrow \frac{1+r}{r} \Pi_j \geq \Pi_d + \frac{1}{r} \Pi_c$$

$$\Rightarrow \boxed{r \le \frac{\pi_j - \pi_c}{\pi_d - \pi_j}}$$

Hence, collusion is easier to sustain, all else equal,

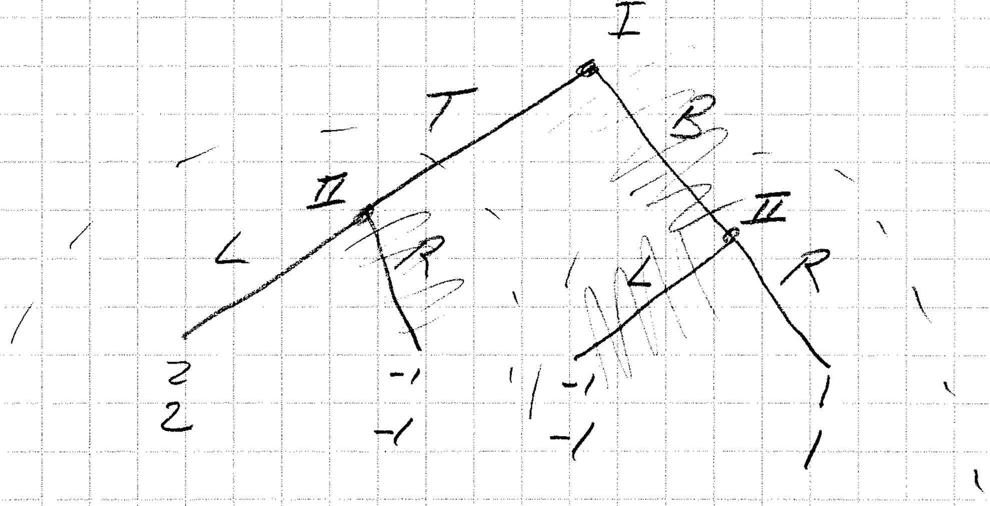
- the smaller the discount rate (r);
- the smaller the net short-term gain from a deviation from collusion ($\pi_d - \pi_j$);
- the larger the net long-term future losses (punishment) from a deviation ($\pi_j - \pi_c$).

15.9 Coordination Game

		II: L	II: R
I	T	$\underline{2}, \underline{2}$ *	$-1, -1$
	B	$-1, -1$	$\underline{1}, \underline{1}$ *

a) The two pure strategy N.E. are (T, L) & (B, R).

d) Sequential game:



The subgame perfect N.E. of this two-stage game is:

II plays L if I plays T.
II " R if I " B.
I plays T.

We have therefore eliminated the N.E. (B, R) in the simultaneous game. The fact that one player moves first allows the players to "coordinate" on a superior N.E.