Q. KR 4.5)

According to Theorem 3.4, the cost fctn of a CRS production technology can be expressed as:

$$c(\vec{w}, y) = y\, c(\vec{w}, 1).$$

If $p(q)$ is the inverse demand, the LR equil. is such that

$$\pi_i = p\left(\sum_{j=1}^{J} y_j\right) y_i - y_i\, c(\vec{w}, 1) = 0, \quad \forall i$$

$$\Rightarrow \left[p\left(\sum_{j=1}^{J} y_j\right) - c(\vec{w}, 1)\right] y_i = 0$$

Hence, the long-run equil. is given by

$$p\left(\sum_{j=1}^{J} y_j\right) = c(\vec{w}, 1).$$

$$\Rightarrow \sum_{j=1}^{J} y_j = q(c(\vec{w}, 1)).$$

The LR equil. is thus characterized by a <u>total</u> output equal to $q(c(\vec{w}, 1))$, regardless of the number of firms producing or the distribution of output across firms.

J.&R. 4.6)

Assumptions:

- Competitive industry.
- total cost: $c_j(q) = aq + b_j q^2$

a) $b_j > 0$.

The competitive equilibrium is characterized by $MC = p$.

$\Rightarrow c_j'(q) = a - 2b_j q_j = p(\sum_i q_i)$

$\Rightarrow q_j = \dfrac{a - p(\sum_i q_i)}{2b_j}$

Since $a - p(\sum q_i)$ is the same for all firms, those with larger b_j will produce less in the competitive equilibrium.

b) $b_j < 0$:

We have:

$\Pi_j = pq_j - (aq_j + b_j q_j^2)$

$\Rightarrow \dfrac{\partial \Pi_j}{\partial q_j} = p - a - 2b_j q_j$

$\dfrac{\partial^2 \Pi_j}{\partial q_j^2} = -2b_j > 0.$

The second-order condition for a maximum is not respected. This means that if marginal profits can be positive, firms will want to keep on producing more.

Note that production costs are nil at

$$a + b_j q_j = 0 \Rightarrow q_j = \frac{-a}{b_j}.$$

Such a production function does not make much sense when $b_j < 0$.

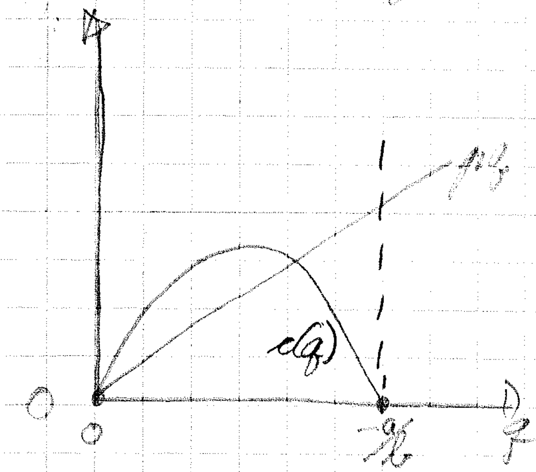
J. & R. 4.7) Assume a competitive industry.

a) $a>0$, $b<0$.

$$\Pi_j = p\,q - (aq + bq^2)$$

$$\Rightarrow \frac{\partial \Pi_j}{\partial q} = p - a - 2bq$$

$$\frac{\partial^2 \Pi_j}{\partial q^2} = -2b > 0 : \text{SOC is violated.}$$



$\Rightarrow$ As long as the price is positive, firms will produce at zero cost, i.e.

$$c(q) = aq + bq^2 = 0 \Rightarrow a + bq = 0 \Rightarrow q = \frac{-a}{b} > 0.$$

$$\Rightarrow Jq = \frac{-Ja}{b}$$

$$\Rightarrow p = \alpha + \frac{\beta J a}{b}$$

If $p = \alpha + \frac{\beta J a}{b} \geq 0$, then we have an equilibrium in the sense that no firm can do better by choosing another output level.

If $p = \alpha + \frac{\beta J a}{b} < 0$, then firms prefer $q = 0$. But this cannot be an equilibrium since with $Jq = 0$, $p = \alpha > 0$ and all firms want to produce. $\Rightarrow$ Problem of existence of competitive equilibrium due to increasing returns to scale.

b) With free entry and exit, we have an equilibrium such that:

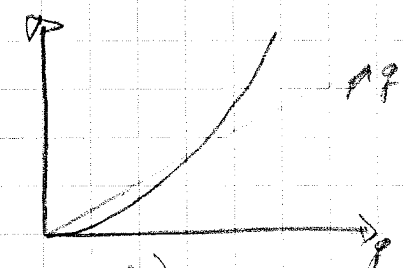
$$q = \frac{-a}{b} \Rightarrow c(q) = 0.$$

$$p = \alpha + \frac{\beta \bar{J} a}{b} = 0.$$

With $p = 0$ and $c(q) = 0$, profits are nil. The equilibrium number of firms is:

$$\bar{J} = \frac{-\alpha b}{\beta a} > 0.$$

c) $a > 0$, $b > 0$.



$$\pi_j = p q_j - (a q_j + b q_j^2) = (p - (a + b q_j)) q_j$$

$$\frac{\partial \pi_j}{\partial q_j} = p - a - 2b q_j = 0 \Rightarrow q_j = \frac{p-a}{2b}$$

$$\frac{\partial^2 \pi_j}{\partial q_j^2} = -2b < 0. \quad \text{SOC is \underline{OK}.}$$

$$\Rightarrow q = \frac{\alpha - \beta J q - a}{2b} \Rightarrow q\left(1 + \frac{\beta J}{2b}\right) = \frac{\alpha - a}{2b}$$

$$\Rightarrow q = \frac{\alpha - a}{2b + \beta J}$$

$$\Rightarrow Jq = \frac{J(\alpha - a)}{2b + \beta J}$$

In the LR, $\pi_j = 0$. Hence,

$$\alpha - \beta(\bar{J}q) - \left[a + b\left(\frac{\alpha - a}{2b+\beta\bar{J}}\right)\right] = 0$$

$$\Rightarrow \alpha - \frac{\beta\bar{J}(\alpha-a)}{2b+\beta\bar{J}} = a + b\left(\frac{\alpha-a}{2b+\beta\bar{J}}\right)$$

$$\Rightarrow \cancel{(\alpha-a)}(2b+\beta\bar{J}) = b(\alpha-a) + \beta\bar{J}(\alpha-a) = \cancel{(\alpha-a)}(b+\beta\bar{J})$$

$$\Rightarrow \quad 2b+\beta\bar{J} = b+\beta\bar{J} \text{ : Impossible unless } \bar{J}\to\infty.$$

This suggests that $\bar{J} \to +\infty$. We have

$$\lim_{J\to\infty} q = \lim_{J\to\infty} \frac{\alpha-a}{2b+\beta J} = 0.$$

$$\lim_{J\to\infty} Jq = \lim_{J\to\infty} \frac{J(\alpha-a)}{2b+\beta J} \underset{\text{l'Hospital rule}}{=} \lim_{J\to\infty} \frac{\alpha-a}{\beta} = \frac{\alpha-a}{\beta}$$

$$\Rightarrow \lim_{J\to\infty} p = \alpha - \beta\left(\frac{\alpha-a}{\beta}\right) = a$$

This is consistent with

$\frac{\partial \Pi_j}{\partial q_j} = 0$ when $q_j \to 0$ and $p = a$.

and $\Pi_j = 0$.

In the long-run free entry and exit equilibrium, the number of firms is arbitrarily large, each produces an infinitesimally small output level, and the price is equal to cost parameter a.

J.&R. 4.27)

Assumptions:

- monopoly
- $q = p^{-\epsilon}$, $\epsilon > 1$. $\Rightarrow p(q) = q^{-1/\epsilon}$
- $tc(q) = cq$
- unit tax $\Rightarrow t(q) = tq$: total tax

Monopoly equil. $\Rightarrow mr(q) = mc(q)$

$tr(q) = p(q)q = q^{-1/\epsilon} \cdot q = q^{1-1/\epsilon}$

$\Rightarrow mr(q) = (1 - \frac{1}{\epsilon})q^{-1/\epsilon} \equiv$ marginal revenue.

With a tax t per unit of output the cost of producing q can be expressed as: $(c+t)q$

The effective marginal cost is thus $t+c$. We thus have:

$mr = mc \Rightarrow (1-1/\epsilon)q_M^{-1/\epsilon} = c+t$

$$\Rightarrow q_M^{-1/\epsilon} = \frac{c+t}{1-1/\epsilon}$$

Since $p = q^{-1/\epsilon}$, we have:

$p_m(t) = \frac{c+t}{1-1/\epsilon} \equiv$ monopoly price.

If $t=0$, we have $p_m(0) = \frac{c}{1-1/\epsilon}$.

Hence, with a unit tax t, the monopoly price increases by

$\frac{t}{1-1/\epsilon} > t$. QED.

J. & R. 4.29)

assumptions:

- Two periods, $t = 1, 2$.
- Monopoly
- $p_t = p(q_t)$
- r = discount rate.

a) $c_t = c(q_t)$:

Problem of firm is to max. present discounted value (PDV):

$$\max_{q_1, q_2} PDV = p(q_1)q_1 - c(q_1) + \left(\frac{1}{1+r}\right)\left[p(q_2)q_2 - c(q_2)\right]$$

FOCs:

$$\frac{\partial PDV}{\partial q_1} = mr(q_1^*) - mc(q_1^*) = 0 \Rightarrow mr(q_1^*) = mc(q_1^*).$$

$$\frac{\partial PDV}{\partial q_2} = \left(\frac{1}{1+r}\right)\left[mr(q_2) - mc(q_2)\right] = 0$$

$$\Rightarrow mr(q_2^*) = mc(q_2^*).$$

Hence, the firm maximizes period-by-period profits. QED

b) Learning-by-doing:

$$c_1 = c_1(q_1)$$

$$c_2 = c_2(q_1, q_2), \quad \frac{\partial c_2}{\partial q_1} < 0.$$

The problem of the firm is:

$$\max_{q_1, q_2} PDV = p(q_1)q_1 - c_1(q_1) + \left(\frac{1}{1+r}\right)\left[p(q_2)q_2 - c_2(q_1, q_2)\right]$$

FOCs:

$$\frac{\partial PDV}{\partial q_1} = mr(q_1^*) - \frac{\partial c_1}{\partial q_1} - \left(\frac{1}{1+r}\right)\frac{\partial c_2}{\partial q_1} = 0. \qquad [1]$$

$$\frac{\partial PDV}{\partial q_2} = \left(\frac{1}{1+r}\right)\left[mr(q_2) - \frac{\partial c_2}{\partial q_2}\right] = 0 \qquad [2]$$

From [1], we have:

$$mr(q_1^*) = \frac{\partial c_1(q_1^*)}{\partial q_1} + \left(\frac{1}{1+r}\right)\frac{\partial c_2(q_1^*, q_2^*)}{\partial q_1} \qquad [1']$$

From [2], we have:

$$mr(q_2^*) = \frac{\partial c_2(q_1^*, q_2^*)}{\partial q_2} \qquad [2']$$

Since $\partial c_2/\partial q_1 < 0$, condition [1'] implies $mr(q_1^*) < \frac{\partial c_1(q_1^*)}{\partial q_1}$.

In order to benefit from the learning-by-doing effect, the firm will produce extra units at a loss during period 1. This loss will be compensated by the lower production costs in period 2. Note that the benefit from lower cost in period 2 must be discounted at rate r.