Jehle 3.53

With this type of the problem, the most crucial part is to properly set-up the problem of the firm. In this particular case, this means that you must write down:

1. All of the firm's choice variables;
2. The firm's objective;
3. The firm's constraints.

In this problem, the firm must choose its input levels: K, F_D, F_N. It cannot choose its output levels y_D and y_N, because they are imposed on the firm. This indicates that the problem of the profit maximizing firm is to minimize costs. Hence the following objective:

$$\min_{K,F_D,F_N} C \;=\; w_K K + w_F(F_D + F_N)$$

The constraints of the firm are that $y_D = 4$ and $y_N = 3$. (Note that we could also add that $y_{TOT} = y_D + y_N = 7$, but this is superfluous.) Given the production function $y_i = \sqrt{KF_i}$, we can write down the firms problem as:

$$\min_{K,F_D,F_N} C \;=\; w_K K + w_F(F_D + F_N) \tag{1}$$

$$\text{s.t. } \sqrt{KF_D} = 4 \text{ and } \sqrt{KF_N} = 3. \tag{2}$$

This problem can be solved using the Lagrangian function. Alternatively, we can use the constraints to substitute directly the following:

$$F_D = 16/K, \tag{3}$$

$$F_N = 9/K. \tag{4}$$

Hence, the cost minimization problem is

$$\min_{K,F_D,F_N} C \;=\; w_K K + w_F(16/K + 9/K). \tag{5}$$

$$\begin{aligned} \min_{K,F_D,F_N} C &= w_K K + w_F(16/K + 9/K) \\ &= w_K K + w_F(25/K). \end{aligned}$$

Taking the FOC, we get

$$K^* = 5\sqrt{\frac{w_F}{w_K}}. \tag{6}$$