J&R 1.42)

Def. of a normal good i: $\Delta^+ y \Rightarrow \Delta^+ x_i$.

(1) Show that if $\Delta^+ y \Rightarrow \Delta^+ x_i$ then $\Delta^- p_i \Rightarrow \Delta^+ x_i$.

PROOF: From the Slutsky eqtn, we have:

$$\frac{\partial x_i}{\partial p_i} = \frac{\partial x_i^h}{\partial p_i} - x_i \frac{\partial x_i}{\partial y} \qquad [*]$$

We also know that the own-substitution effect must be negative; i.e.

$$\frac{\partial x_i^h}{\partial p_i} \leq 0.$$

Hence, if $\frac{\partial x_i}{\partial y} > 0$, the RHS of [*] must be negative. QED

(2) Show that if $\Delta^+ p_i \Rightarrow \Delta^+ x_i$ then $\Delta^+ y \Rightarrow \Delta^- x_i$.

PROOF: Since $\frac{\partial x_i^h}{\partial p_i} \leq 0$, the Slutsky eqtn [*] tells that with $\frac{\partial x_i}{\partial p_i} > 0$, it must be the case that $\frac{\partial x_i}{\partial y} < 0$. QED

(3) Show that if $\Delta^- p_i \Rightarrow \Delta^+ x_i$ then it is not necessarily the case that $\Delta^+ y \Rightarrow \Delta^+ x_i$.

Let $\frac{\partial x_i}{\partial y} < 0$ while $x_i \frac{\partial x_i}{\partial y} < -\frac{\partial x_i^h}{\partial p_i}$. Then

$$\frac{\partial x_i}{\partial p_i} < 0. \qquad \text{QED}$$

(4) Show that if $\Delta^+ y \Rightarrow \Delta^- x_i$, it is not necessarily the case that $\Delta^+ p_i \Rightarrow \Delta^+ x_i$.

Assume that $\frac{\partial x_i}{\partial y} < 0$ while

$-x_i \frac{\partial x_i}{\partial y} < -\frac{\partial x_i^u}{\partial p_i}$. Then, according to the Slutsky eqtn. we have

$\frac{\partial x_i}{\partial p_i} < 0$. QED