EXERCISES SET 5
PRODUCTIVITY, TECHNOLOGY AND EFFICIENCY

(1) As seen in class, we suppose that the level of total factor productivity $\bar{A}$ depends on the levels of technology T and efficiency E in the following manner $\bar{A} = T \times E$. We observe that the productivity level in Country X is twice as high as that of Country Z. If the technology level in Country X is four times that of Country Z, how do the efficiency levels of the two countries compare?

According to the question, we have the following data: $A_X = 2A_Z$ and $T_X = 4T_Z$. If we assume that productivity depends on efficiency and technology as per the following equation $A = T \times E$, the data implies that

$$T_X \times E_X = 2 \times T_Z \times E_Z, \tag{1}$$

$$\Rightarrow 4 \times T_Z \times E_X = 2 \times T_Z \times E_Z, \tag{2}$$

$$\Rightarrow E_X = \frac{2 \times E_Z}{4} = \frac{E_Z}{2}. \tag{3}$$

Country Z is twice as efficient as Country X.

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(2) Relative to Canada, the productivity level in Country X is 0.5. The growth rate of technology is 1% per year. What is the level of efficiency in Country X relative to Canada if Country X lags behind Canada by 20 years in terms of technology?

According to the data, we have $A_{X,2007} = 0.5 \times A_{CAN,2007}$ and $T_{X,2007} = T_{CAN,1987}$. Also, technology grows at rate $g = 1\%$ per year. As a result, we have

$$T_{CAN,2007} = T_{X,2007}(1.01)^{20} \tag{4}$$

$$\Rightarrow \frac{T_{X,2007}}{T_{CAN,2007}} = (1.01)^{-20} = 0.82, \tag{5}$$

and thus

$$\frac{A_{X,2007}}{A_{CAN,2007}} = \frac{T_{X,2007}}{T_{CAN,2007}} \times \frac{E_{X,2007}}{E_{CAN,2007}}, \tag{6}$$

$$\Rightarrow 0.5 = 0.82 \times \frac{E_{X,2007}}{E_{CAN,2007}}, \tag{7}$$

$$\Rightarrow E_{X,2007} = 0.61 \times E_{CAN,2007}. \tag{8}$$

The efficiency level in Country X is 61% that of Canada.

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(3) We have seen that the productivity level in India is 0.35 that of the USA. Suppose that efficiency levels in both countries are the same and that the growth rate of productivity in the USA is 0.81% per year. Calculate how many years India lags behind the USA?

We assume that $E_{INDIA} = E_{USA}$ and that all of productivity growth in the USA is due to technological progress only. We have $A_{INDIA,98} = 0.35 \times A_{USA,98}$. Moreover,

$$\frac{A_{INDIA,98}}{A_{USA,98}} = \frac{T_{INDIA,98}}{T_{USA,98}} \times \frac{E_{INDIA}}{E_{USA}} = \frac{T_{INDIA,98}}{T_{USA,98}}. \tag{9}$$

Hence, $T_{INDIA,98} = 0.35 \times T_{USA,98}$.

Now if technology grows at rate 0.81% per year, we have

$$T_{USA,98} = T_{USA,98-G}(1.0081)^G \tag{10}$$

$$\Rightarrow T_{USA,98-G} = \frac{T_{USA,98}}{(1.0081)^G}. \tag{11}$$

We are thus looking for the value of G such that

$$T_{USA,98-G} = \frac{T_{USA,98}}{(1.0081)^G} = 0.35 \times T_{USA,98} \tag{12}$$

$$\Rightarrow \frac{1}{0.35} = (1.0081)^G, \tag{13}$$

$$\Rightarrow \ln \frac{1}{0.35} = G \times \ln 1.0081, \tag{14}$$

$$\Rightarrow G = 130. \tag{15}$$

The upshot is that if India and the USA were equally efficient, India would have to be 130 years behind the USA in its technology level in order to explain its lower productivity level. Clearly, such a technological lag is unlikely to be true. We thus conclude that India must be less efficient in its use of factors and technology than the USA.

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(4) In capital cities, urban workers often have more "political clout" than rural workers. Suppose that this allows the urban workers to institute a minimum wage level in the city. This minimum wage is above the one that would prevail if labor wages were set perfectly competitively and workers could move freely between the urban and rural sectors. Show why this can create an inefficient allocation of labor between the two sectors. Who are the main losers from the higher urban minimum wage?

(The accompanying graphic to this answer is in file `graph-rural-urban-efficiency.pdf`.)

An efficient allocation of labor between the two sectors is represented by quantities L_R^* and L_U^*. This is because marginal productivities are equal between sectors and there is no unemployment, i.e. $L_R^* + L_U^* = \bar{L}$. In a competitive labor market, the wage is equal to the marginal productivity of labor at level w^*.

We introduce a minimum wage in the urban sector, say at level $\underline{w} > w^*$. Since firms are not willing to hire workers with marginal productivity below that of the wage rate, no more than L_U' workers will be hired in the urban sector. The rest will work in the rural sector (assuming no unemployment). In the new equilibrium with the minimum urban wage, we see that more workers stay in the rural sector and they receive a lower wage w' than *both* the urban minimum wage and the competitive wage w^*. The urban workers' gain from the minimum wage has a counterpart in terms of lower wages in the rural sector. The minimum wage creates an inefficiency because the marginal productivity of workers is not equalized between sectors.