SOLUTIONS CHAPTER 9
TECHNOLOGICAL PROGRESS

9.1 According to equation (9.3) in the textbook, assuming no growth in land and capital, productivity growth is given by

$$\hat{A} = \hat{y} + \beta \hat{L}.$$

Since income per capita is also assumed constant, while $\beta = 1/3$, we have $\hat{A} = (1/3)\hat{L}$. Now $\hat{L}$ is given by

$$170m = 4m(1 + \hat{L})^{10{,}000}.$$

$$\hat{L} = (170/4)^{1/10000} - 1 = 0.0375\% \text{ per year.}$$

The average productivity growth rate is thus estimated to be

$$\hat{A} = (1/3)(0.0375\%) = 0.0125\% \text{ per year.}$$

9.2 A higher productivity in the wheat sector implies that less inputs are necessary to produce a unit of output. Meanwhile, the absence of technological progress in the hair cutting business means that the same number of inputs are still required per unit of output. Assuming that inputs are paid at the same rate in both sectors (as would be the case if the markets for inputs and outputs were truly competitive), the relative price of wheat goes down relative to haircuts.

The second question is actually more complicated than first meets the eye. Let us first suppose that there is *no movement of labor* between both sectors. As technology improves in the wheat sector, the productivity of farmers goes up; that is, each produces more wheat than before. This tends to increase their income relative to barbers. But at the same time, as there is more wheat around, the lower relative price of wheat means that barbers receive more wheat per haircut. So barbers are unambiguously better off. Whether farmers are better off or not depends on the price effect, which in turn depends on the demand schedules for both wheat and haircuts. Imagine, for instance, that nobody is really interested in buying the extra wheat being produced (inelastic demand). As farmers compete to sell their wheat, the price of wheat can become so low that farmers are made worse off; that is, the proportional increase in price for a haircut exceeds the proportional increase in output that resulted from the productivity improvement. Conversely, if the demand for wheat is very elastic, then the price is almost unaffected by the larger quantity being supplied. Then farmers should benefit more than barbers.

If there is a *free movement of labor* between both sectors, then we would expect all workers to receive the same income, with income equal to the marginal product of labor. In such a case, since more output is being produced overall, all will be made equally better off in the end. This examples underscores the important role played by the free movement of inputs between sectors, among other things.

9.3 Technological progress in the production of a particular good will reduce the price of that good. The reduction in price of the good will in turn change the quantity demanded for that good, usually an increase. There are thus two opposite effects in terms of expenditures: for a given quantity, a lower price reduces the share spent on the good; for a given price, a higher quantity increases the share spent on the good. Which of the two effects prevails depends on the price-elasticity of demand for that good.

If the price-elasticity of demand for the good is large, a reduction in its price of a good will result in a large increase in its demand, thus leading to a higher expenditure share on that good.

Conversely, if the price-elasticity of demand is small, a reduction in the price of a good will result in a small increase in its demand, thus leading to a lower expenditure share on that good.

Technological advance determines the change in the price, and the price-elasticity of demand determines the subsequent change in expenditure share on the good.

9.4)

a) Since each slice of bread must be consumed with one slice of cheese, we must have $Y_b = Y_c$.

$$\Rightarrow A_b L_b = A_c L_c$$

Insert the fact that $L_c = \bar{L} - L_b$ to get

$$A_b L_b = A_c(\bar{L} - L_b)$$

$$\Rightarrow L_b = \frac{A_c \bar{L}}{A_b + A_c} \quad [*]$$

In year 2000, we have $A_b = A_c = 1$. Hence

$$L_b = \frac{\bar{L}}{2} \quad \text{and} \quad L_c = \frac{\bar{L}}{2}$$

Each sector employs half of the workforce.

b) Given $\hat{A}_b = 2\%$ and $\hat{A}_c = 1\%$, in year 2001 we will have $A_b = 1.02$ and $A_c = 1.01$. Since $Y_b = Y_c$ and $L_b + L_c = \bar{L}$ must both still hold in 2001, expression [*] above gives:

$$L_b = \frac{1.01}{1.02 + 1.01}\bar{L} = 0.4975\bar{L}$$

$$\Rightarrow Y_{b2001} = 1.02 \cdot 0.4975\bar{L} = 0.5075\bar{L} = Y_{c2001}$$

In year 2000, we have

$$Y_{b2000} = 0.5\bar{L} = Y_{c2000}.$$

This gives a growth rate of output equal to:

$$\vec{Y}_{b2000} = \frac{Y_{b2001} - Y_{b2000}}{Y_{b2000}} = \frac{0.5075\bar{L} - 0.5\bar{L}}{0.5\bar{L}}$$

$$= 1.5\%.$$

1.0404 1.0201

2.0605

c) In year 2002, we have

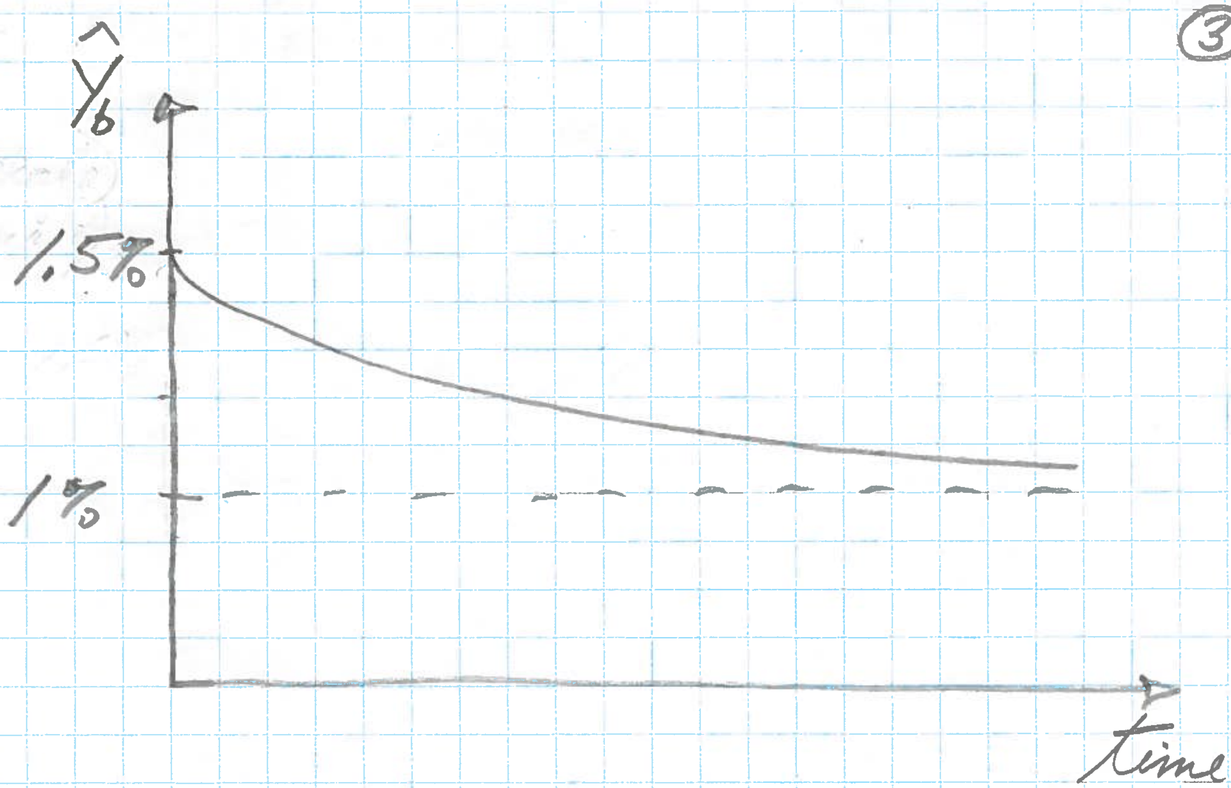
$$L_{b2002} = \frac{(1.01)^2}{(1.02)^2 + (1.01)^2}\bar{L} = 0.4951\bar{L}$$

$$\Rightarrow Y_{b2002} = (1.02)^2 \cdot 0.4951\bar{L} = 0.515075\bar{L}$$

$$\Rightarrow \vec{Y}_{b2001} = \frac{0.515075 - 0.5075}{0.5075} \doteq 1.493\%$$

We see that as time goes by, the labor force moves out of the higher-growth sector and into the lower-growth one. This causes the growth rate to decrease over time.

Eventually, as most of the labor force is employed in the low-growth sector, the growth rate will be equal to 1%.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