SS OUTPUT AND TECH. PROGRESS

a) In SS, we must have $\Delta k_e = 0$ where $k_e = \frac{K}{eL}$.

$$\Delta k_e = \gamma k_e^{1/2} - (\delta + n + \hat{e}) k_e = 0$$

$$\Rightarrow k_e^{ss} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^2 = \left(\frac{0.16}{0.1 + 0.02 + 0.04}\right)^2$$

i) $\Rightarrow \boxed{k_e^{ss} = 1}$

ii) $\Rightarrow y_e^{ss} = \left(k_e^{ss}\right)^{1/2} = 1$

iii) $\hat{y}_e^{ss} = 0$ by definition of the SS since $\hat{k}_e^{ss} = 0$.

iv) $y_e = \frac{y}{e} \Rightarrow \hat{y}_e = \hat{y} - \hat{e}$

Since $\hat{y}_e^{ss} = 0$, we have $\hat{y}^{ss} = \hat{e} = 4\%$.
Per capita income grows at a rate of 4% per year.

v) $y_e = \frac{Y}{eL} \Rightarrow \hat{y}_e = \hat{Y} - \hat{e} - n$

In SS: $\hat{Y}^{ss} = \hat{e} + n = 4\% + 2\% = 6\%$

Aggregate output grows at a rate of 6% per year in the long run.

vi) $y_{et} = \frac{Y_t}{e_t} \Rightarrow Y_t = e_t\, y_{et}$

$\Rightarrow y_t^{ss} = e_t\, y_e^{ss} = e_t \cdot 1$

$\Rightarrow \boxed{y_t^{ss} = e_t}$

b) $\underline{\hat{e} = 8\%}$

$$\Rightarrow k_e^{ss} = \left(\frac{0.16}{0.1+0.02+0.08}\right)^2 = 0.64$$

$$\Rightarrow y_e^{ss} = (0.64)^{1/2} = 0.8$$

$$\Rightarrow \hat{y}_e^{ss} = 0$$

$$\Rightarrow \hat{y}^{ss} = 8\%$$

$$\hat{Y}^{ss} = 8\% + 2\% = 10\%, \quad \boxed{y_t^{ss} = 0.8e_t}$$

The increase in growth of TP actually reduces capital stock and income level per effective worker in the long run. But this does not mean that income per capita is reduced. In fact, in the long run, it now grows at a higher rate of 8% per year, while aggregate output growth increases to 10%.

c) $\hat{e} = 4\%$, $n = 6\%$.

$$\Rightarrow k_e^{ss} = \left(\frac{0.16}{0.1 + 0.06 + 0.04}\right)^2 = 0.64$$

$$\Rightarrow y_e^{ss} = (0.64)^{\frac{1}{2}} = 0.8$$

$\hat{y}_e^{ss} = 0$ by def.

$$\Rightarrow \hat{y}^{ss} = 4\%$$

$$\hat{Y}^{ss} = 4\% + 6\% = 10\%.$$

$$\boxed{y_t^{ss} = 0.8 e_t}$$

d) In the long run, levels of income per capita are:

$y_{at} = e_{at}$ (for a))

$y_{bt} = 0.8\, e_{bt}$ (for b))

$y_{ct} = 0.8\, e_{ct}$ (for c))

where $e_{at} = e_{ct} < e_{bt}$.

Hence: $y_{at} > y_{ct}$

all else equal

People are clearly worse off with a higher population growth even though aggregate output grows faster.

Also: $y_{bt} > y_{ct}$

Even though the SS output per effective worker are the same in (b) and c), per capita income levels will be higher with higher TP growth than population growth, as would be expected.

Note, however, how the aggregate output growth is the same for both b) and c), though for different reasons.

Finally, we need more mathematical derivation to compare the magnitudes of y_t^a and y_t^b. It can be shown that $y_t^b > y_t^a$, as would be expected by intuition, i.e. a higher TP growth rate increases per capita income, all else equal.