WORKING WITH GROWTH RATES

Suppose we observe some economic quantity X in two adjacent years. The growth rate of X is the change in X from the first year to the second, divided by the value of X in the first year. Let t designate the first year and $t + 1$ designate the second. Mathematically, if we call the observations X_t and X_{t+1}, the growth rate g is given by this equation:

$$g = \frac{X_{t+1} - X_t}{X_t}.$$

For example, if $X_t = 100$ and $X_{t+1} = 105$, then the annual growth rate is

$$g = \frac{105 - 100}{100} = 5/100 = 0.05 = 5\%.$$

We can modify this formula to find an average rate of growth over several years. First rewrite the formula for the growth rate as follows:

$$X_{t+1} = X_t \times (1 + g).$$

Now consider the case where X grows at the same rate, g, for two years in a row. Rewrite the equation to apply to years $t + 1$ and $t + 2$, then substitute for X_{t+1} from the same equation:

$$\begin{aligned} X_{t+2} &= X_{t+1} \times (1 + g) \\ &= [X_t \times (1 + g)] \times (1 + g) \\ &= X_t \times (1 + g)^2. \end{aligned}$$

Similarly, if something grows at rate g for n years, we can write

$$X_{t+n} = X_t \times (1 + g)^n.$$

Suppose now that X_t and X_{t+n} are known. We can rearrange our previous equation, solving for g to obtain the average growth rate (technically, this is the geometric average growth rate) over this time:

$$g = \left(\frac{X_{t+n}}{X_t}\right)^{1/n} - 1.$$

For example if we observe $X_t = 100$ and $X_{t+20} = 200$, then the average rate of growth is

$$\begin{aligned} g &= \left(\frac{200}{100}\right)^{1/20} - 1 \\ &= 1.035 - 1 = 0.035 = 3.5\%. \end{aligned}$$

To graph data on variables that grow over time, it is often useful to employ a ratio scale (also called a logarithmic scale). On a **ratio scale,** equal spaces on the vertical axis correspond to equal *proportional* differences in the variable being graphed. For example, the vertical gap between $X = 1$ and $X = 10$ is the same as the vertical gap between $X = 10$ and $X = 100$. (By contrast, on the more common **linear scale,** equal spaces on the vertical axis correspond to equal differences in the variable being graphed.) On a ratio scale, a quantity growing at a constant rate plotted over time will yield a straight line. Figure 1.3 shows an example of how a ratio scale changes our perspective. Both panels consider some quantity X that starts with a value of 1 in the year 0 and grows at a rate of 3% per year for 200 years. The upper panel uses a linear scale, and the lower panel uses a ratio scale.

A useful mathematical approximation for dealing with growth rates is the **rule of 72.** The "rule" is a formula for estimating the amount of time it takes something growing at a given rate to double:

$$\text{doubling time} \approx \frac{72}{g},$$

where g is the percentage annual growth. For example, if something is growing at 2% per year, then it will double in approximately 36 years.

growth rate of GDP per capita over this period was 1.9% per year. Such a change is hardly noticeable from one year to the next. But compounded over 135 years, the effect has been dramatic.

When we look at data on income over long periods of time, it is often useful to use a ratio scale (see "Working with Growth Rates"). Figure 1.4 uses a ratio scale to examine the same data as we examined in Figure 1.2: GDP per capita in the United States. In Figure 1.4, note how regular the process of growth appears when viewed over such a long horizon. The year-to-year fluctuations in output

The Effect of Usi

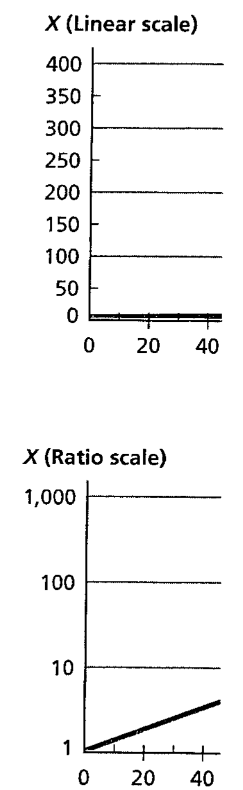

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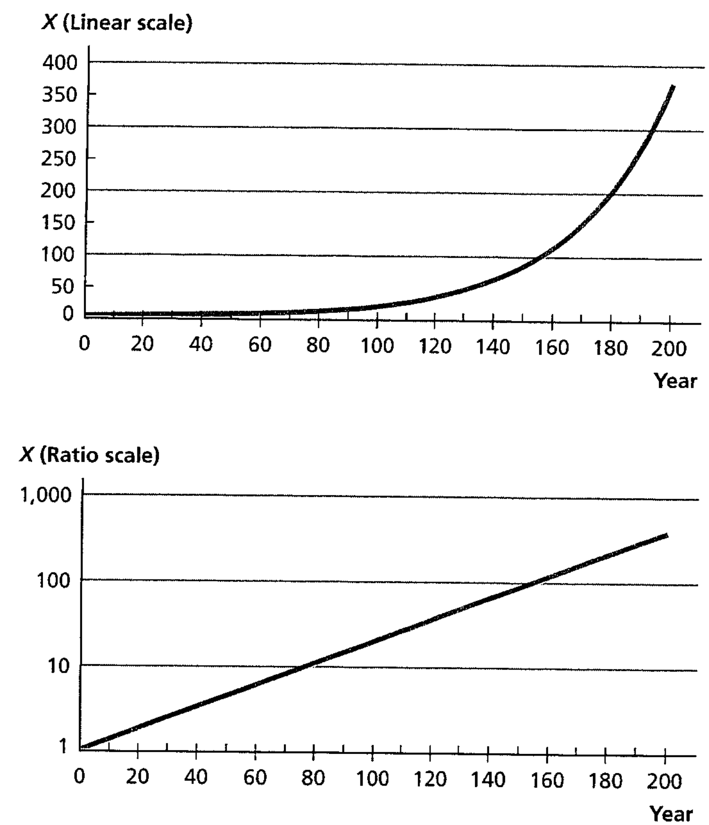
FIGURE 1.3

The Effect of Using a Ratio Scale



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remarkably predictable. For example, a forecast made in 1929 simply by drawing a trend line through the data up to that year would come within a small margin (17%) of accurately forecasting output per capita 70 years later.

The experience of the United States over this period explains the way that Americans have come to think about growth. Presidential candidates ask, "Are you better off than you were four years ago?" implying that getting better off all the time is the natural state of affairs. It turns out, however, that the experience of constant,

QUESTIONS FOR REVIEW

1. What is the magnitude of income differences between the richest and poorest countries in the world today?
2. What is the magnitude of income differences between the world's richest countries today and their own income per capita 200 years ago?
3. In what cases is GDP per capita the best measure of a country's income? In what cases is total GDP the best measure?
4. How does the average growth rate of income per capita in the world since 1960 compare with growth in the previous century? How did growth in the 19th century compare with growth in the previous centuries?
5. What is the relative importance of within-country inequality and between-country inequality in explaining total world income inequality? How has the relative importance changed over time? Why?

PROBLEMS

1. How would using a ratio scale (rather than a linear scale) affect Figure 1.1?
2. How fast would a country have to be growing in order to double its output in nine years? You should answer this question using the rule of 72, *not* a calculator.
3. Suppose that in a particular country, GDP per capita was $1,000 in 1900 and $4,000 in 1948. Using the rule of 72 (*not* a calculator), approximate the annual growth rate of GDP per capita.
4. Suppose that the entire population of the world consists of four people, divided into two countries of two people each. The following table shows data on their income and nationality. Based on this table, which is the more important source of world inequality: between-country inequality or within-country inequality?

Person	Nationality	Income
Alfred	Country A	1,000
Bob	Country B	2,000
Carol	Country B	3,000
Doris	Country A	4,000

5. In 1900 GDP per capita in Japan (measured in year 2000 dollars) was $1,433. In 2000 it was $23,971. Calculate the growth rate of income per capita in Japan over this period. Now suppose that Japan grows at the same rate for the century following 2000. What will Japanese GDP per capita be in the year 2100?

6. In 2005 GDP per capita in the United States was $
capita in Sri Lanka was $4,650. Suppose that incor
States has been growing at a constant rate of 1.9%
that this is roughly true.) Calculate the year in wh
United States was equal to year 2005 income per c
7. Between 1970 and 2005, China's GDP per capita g
7.3% per year while GDP per capita in the United
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6. In 2005 GDP per capita in the United States was $36,806 while GDP per capita in Sri Lanka was $4,650. Suppose that income per capita in the United States has been growing at a constant rate of 1.9% per year. (Figure 1.4 shows that this is roughly true.) Calculate the year in which income per capita in the United States was equal to year 2005 income per capita in Sri Lanka.
7. Between 1970 and 2005, China's GDP per capita grew at an average rate of 7.3% per year while GDP per capita in the United States grew at an average rate of 2.2%. In 2005, U.S. GDP per capita was $36,806 and Chinese GDP per capita was $5,955. Assuming that the two countries continue to grow at these rates, in what year will China overtake the United States in terms of GDP per capita?

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g PPP on Comparisons of GDP

GDP per Capita in 2005 Using Market Exchange Rates (dollars)	GDP per Capita in 2005 Using PPP Exchange Rates (dollars)
37,267	37,267
39,075	27,817
23,906	26,210
8,094	12,704
6,172	9,564
588	3,072

ımed worldwide). Such a basket would have a price of 30 dollars 20 dollars in Poorland. The prices of the basket in the two a purchasing power exchange rate of two Poorland dollars for nd dollars. Using this exchange rate, Poorland's GDP per capita ars) would be worth 30 Richland dollars—and we would con- that, on the basis of PPP exchange rates, Poorland's GDP per arter of Richland's.

his book we will use PPP exchange rates in making comparisons Table 1.3 shows the effect of switching from market exchange ınge rates for a typical set of countries.

shows, Japan's currency was overvalued relative to PPP; using market exchange rates lowers Japan's level of GDP per capita. ᴾP exchange rates makes Japan look poorer than the United ısing market exchange rates makes Japan look richer. Mexico typical for developing countries, had market exchange rates y undervalued relative to PPP. In the case of India, for exam- PPP raises GDP per capita relative to the United States by a

e rates are useful for comparing quantities other than GDP. For listic reports of conditions in developing countries will often erage wage of a worker into U.S. dollars using the market sing a PPP exchange rate instead would yield a different, more

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r of people worldwide living on less than one dollar per day can

A.2. Suppose that there are only two goods produced in the world: computers, which are traded internationally, and ice cream, which is not. The following table shows information on the production and prices of computers and ice cream in two countries:

a. Calculate the level of GDP per capita in each country, measured in its own currency.

Country	Computers Produced per Capita	Ice Cream Produced per Capita	Price of Computers in Local Currency	Price of Ice Cream in Local Currency
Richland	12	4	2	4
Poorland	3	1	1	1

b. Calcuate the market exchange rate between the currencies of the two countries.

c. What is the ratio of GDP per capita in Richland to GDP per capita in Poorland, using the market exchange rate?

d. Calculate the purchasing power parity (PPP) exchange rate between the two currencies.

e. What is the ratio of GDP per capita in Richland to GDP per capita in Poorland, using the PPP exchange rate?

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